MAR513: Homework # 5

Consider the simple radiation boundary condition

$$\frac{\partial \phi}{\partial t} + c\frac{\partial \phi}{\partial x} = 0,$$

where c is constant.

a. Write down the forward time/backward space finite-difference form of the boundary condition;

b. For a wave of frequency ω and wave number k incident upon the open boundary at x =0, find the reflection coefficient for the boundary condition;

c. Show that for $c = 0$ and $c = \infty$, the boundary is a perfect reflector;

d. Show (graphically or other ways) that the magnitude of the reflection coefficient is a minimum when $\omega / k = c$.