

MAR513: Homework#4

1. Examine the stability of the forward time/ centered space scheme applied to the three-dimensional diffusion equation

$$\frac{\partial \phi}{\partial t} = \alpha \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) + \beta \frac{\partial^2 \phi}{\partial z^2} .$$

For $\Delta x = \Delta y = \Delta z$ and $\alpha = \beta$, compare to the one-dimensional case.

2. Consider the Adams-Bashforth scheme applied to the linear equation

$$\frac{\partial \phi}{\partial t} = L(\phi)$$

which is

$$\frac{\phi^{n+1} - \phi^n}{\Delta t} = \frac{3}{2} L(\phi^n) - \frac{1}{2} L(\phi^{n-1}) .$$

a. Show that the scheme is accurate to $O(\Delta t^2)$.

b. Examine the stability of the scheme with $L(\phi) = K \frac{\partial^2 \phi}{\partial x^2}$.