The Performance of a Coupled 1-D Circulation and Bottom Boundary Layer Model with Surface Wave Forcing

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Abstract
This is a numerical study of the combined influence of wave and current bottom stresses on the coastal ocean response. Several experiments were conducted with a coupled bottom boundary layer model (BBLM) with the Dartmouth one-dimensional (1-D) finite element dynamic model NUBBLE. The numerical calculations show that (1) the quadratic drag law simulates the bottom stress for deep water reasonably well; (2) the combined wave and current influence can be very important on bottom stress calculations in shallow waters with depths<80m); (3) larger equivalent drag coefficients are generated by combined wave and current interactions than with the simple sum of both forcing processes.

1. Background

Many models, like the 1-D finite element NUBBLE (Naimie 1995) and the 3-D finite-element QUODDY (Lynch et al. 1996), define bottom stress in terms of a simple quadratic drag law in which the bottom stress is the product of a constant bottom drag coefficient and the square of a near bottom current. However Grant and Madsen (1979) and others have shown that in shallow water the effective bottom stress depends on many factors including sediment type, bed forms, "steady" current speed, oscillatory wave currents and as have shown their nonlinear interaction. Thus it can be important at times of storms to incorporate the time-dependent, spatially varying character of the bottom drag coefficient into the hydrodynamic momentum equations for a coastal ocean area. Grant and Madsen (1979) developed an approach, later refined by Glenn and Grant (1987) and Keen and Glenn (1994), to account for time varying nonlinear effects on bottom stress in shallow water, where surface wave velocities can be significant.

In this paper, we explore how a coupled numerical hydrodynamic 1-D circulation model and bottom boundary layer model can more accurately simulate the effects of the nonlinear interaction between current and prevailing surface gravity waves on local bottom stress. In anticipation of the application of the 3-D finite element numerical model QUODDY to simulate the storm response of the Gulf of Maine, we focus on typical storm wave conditions there. Specifically National Data Buoy Center buoy measurements in the Gulf of Maine document the temporal variability of the significant wave height at the dominant storm wave
period of about 12 seconds. Surface waves of this period interact with the bottom in water depths less than about 80m.

2. The Models

A. NUBBLE

In this application, we couple BBLM with a one-dimensional circulation model called NUBBLE with the Styles (1999) bottom boundary layer model (BBLM). NUBBLE is a 1-D, time-stepping model with linear finite elements and a Mellor and Yamada (1982) level 2.5 turbulence closure to determine the vertical structure of the horizontal components of velocity \((u,v)\) and density \((\rho)\) under specified surface forcing. The governing equations for NUBBLE are the linearized shallow water equations (Naimie, 1995) (Appendix A).

The NUBBLE bottom stress \(\tau_b\) is given as a sink of horizontal momentum in an unstratified flow according to

\[
\kappa_m \frac{\partial V}{\partial z} = \frac{\tau_b}{\rho_0} = C_d |V_b| V_b ,
\]

where \(z\) is the elevation, \(V\) is the depth-dependent horizontal velocity vector, \(\kappa_m\) is the vertical momentum diffusivity, \(C_d\) is the bottom drag coefficient, \(V_b\) is the bottom velocity and \(\rho_0\) is water density.

We ran the model with a vertical elevation discretization defined by

\[
z(\varepsilon) = -h + \varepsilon h - \alpha \sin(2\pi \varepsilon) ,
\]

where \(\varepsilon\) increases linearly with node number from 0 at the bottom to 1 at the free surface and \(h\) is depth. The water column was separated into 20 layers, so that \(\varepsilon = \frac{1}{20}\) and correspondingly

\[
\alpha = (h - 20 \ast DZBL) / (20 \ast \sin(2\pi / 20))
\]

is resolved to obtain the desired nodal spacing (DZBL) at the extremes of the water column. Thus,

\[
z(j) = -h \ast \left(1 - \frac{j-1}{20}\right) - \alpha \ast \sin\left(\frac{2\pi (j-1)}{20}\right) ,
\]
where $h$ is water column depth, $j$ is layer number, $z = -h$ at bottom and $z = 0$ at surface. The resulting mesh is plotted in Figure 1.

Figure 1 (III.1.1) The vertical discretization of the NUBBLE grid.

**B. BBLM**

The bottom boundary layer model that we used was developed by Styles and Glenn (2000) under the guidance of S. Glenn. The model consists of three different zones in which the friction physics differs (Figure 2). Combined wave/current interactions dominate in the lowest (i.e. near-bottom) layer; Current-induced friction dominates in the layer furthest from the bottom. This model is distinguished from the Grant & Madsen (1979) by having a transition layer between the other two layers.

Figure 2. Bottom boundary layer model geometry for the Styles (1998) model, which has three boundary layer regions; one for combined wave and current $0 \leq z \leq z_w$; one for the transition layer $z_w \leq z < z_c$; and one for current only $z > z_c$. The horizontal currents $u_1$ and $u_2$ at elevations $z_1$ and $z_2$ respectively are used to define the near-bottom stress.

This BBLM requires inputs of (1) the “slowly” varying current at a near bottom reference depth (in this case from NUBBLE) and (2) the instantaneous maximum surface wave-induced bottom water particle excursion and velocity amplitudes. Thus the BBLM outputs a near bottom eddy viscosity and a current profile from which the bottom stress can be estimated. The estimated bottom stress is then converted to an equivalent bottom drag coefficient that can be used in NUBBLE.

**3. Model Experiments**

The purpose of these experiments was to diagnose the influence of surface wave-induced bottom stress on NUBBLE flow. First, we conducted a set of reference NUBBLE runs using a set of constant bottom drag coefficients. Second we ran NUBBLE with time-dependent bottom drag coefficients that were generated by the BBLM under different waves and different water depths.

We started each model run by linearly increasing the surface eastward ($x$-directed) geostrophic current $U_g$ to a constant value of 0.3m/s during the first 1000 2-minute time steps. The results of the different experiments are presented next.
4. Results

A. Constant Drag Coefficient Results

We ran a set of NUBBLE experiments with differing constant bottom drag coefficients; $C_d = 0.003, 0.004, 0.005, 0.01, \text{ and } 0.015$ to equilibration after about 500 time steps. The bulk results (Table 1) confirm our expectations that increased bottom stress leads to decreased total water column momentum.

<table>
<thead>
<tr>
<th>Drag Coefficient</th>
<th>0.003</th>
<th>0.004</th>
<th>0.005</th>
<th>0.010</th>
<th>0.015</th>
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<td>X-momentum</td>
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<td>30195</td>
<td>30128</td>
<td>29865</td>
<td>29674</td>
</tr>
<tr>
<td>Y-momentum</td>
<td>1295</td>
<td>1534</td>
<td>1737</td>
<td>2432</td>
<td>2871</td>
</tr>
<tr>
<td>Total momentum</td>
<td>30293</td>
<td>30234</td>
<td>30178</td>
<td>29964</td>
<td>29813</td>
</tr>
<tr>
<td>X-Bottom Stress</td>
<td>0.13</td>
<td>0.15</td>
<td>0.17</td>
<td>0.24</td>
<td>0.28</td>
</tr>
<tr>
<td>Y-Bottom Stress</td>
<td>0.004</td>
<td>0.006</td>
<td>0.008</td>
<td>0.016</td>
<td>0.023</td>
</tr>
<tr>
<td>Total Bottom Stress</td>
<td>0.13</td>
<td>0.15</td>
<td>0.17</td>
<td>0.24</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Table 1. Water column total momentum (kg/m/s) and bottom stress (kg/m$^2$s$^{-2}$) for different constant drag coefficients in 100m of water.

Velocity profiles (upper left panels Figure 3) show that friction-induced effects extend from the bottom to the top of the water column. Interestingly, the eastward surface current exceeds that of the imposed surface eastward geostrophic flow of 0.3 m/s - by an increased amount for cases with higher bottom stress (i.e. lower momentum). Apparently the turbulence closure scheme “rearranges” of the velocity profile such that high surface currents result.

Figure 3 (III.2.1). NUBBLE results for the different constant bottom drag coefficient cases. $Q_2$ is twice the total kinetic energy; $ENZM$ the vertical eddy viscosity (m$^2$/s); $ENZH$ the vertical heat diffusivity (m$^2$/s); $ENZQ$ the vertical diffusivity for turbulence quantities (m$^2$/s);

B. Coupled NUBBLE / BBLM Results: Wave Amplitude Sensitivity

In the real ocean, surface gravity waves are produced where wind stress acts on the ocean. These waves then can propagate into regions where there is no local wind generated waves. To simplify our calculation, we assumed that the specified surface gravity waves were generated somewhere west of our model domain and then propagated eastward into our study region without modification.
These eastward propagating surface waves were aligned with the surface geostrophic current forcing.

During the 1000 time step eastward geostrophic flow ramp-up, a constant Cd = 0.005 was used for the NUBBLE/BBLM model. Starting at time step 1001 the NUBBLE bottom current, the maximum eastward wave bottom velocity and the maximum excursion amplitude were input into the BBLM to obtain a modified bottom stress. The corresponding drag coefficients were derived according to the following scheme.

- The BBLM shear velocity under combined wave current conditions $u_{cw}$ was used to calculate an eddy viscosity $\mu$ according to
  \[ \mu = \kappa u_{cw} \left( z_1 + z_2 \right) / 2 \]
  where $z_1$, $z_2$ are near bottom elevations (see Figure 2), $\kappa$ is the von Karman constant;

- The eddy viscosity in (3) was used to calculate the BBLM bottom stress $\tau$ according to
  \[ \tau = \mu \frac{u_2 - u_1}{z_2 - z_1} \]
  where $u_1$, $u_2$ are currents at the near bottom elevations $z_1$, $z_2$.

- The corresponding “new” NUBBLE drag coefficient is
  \[ C_d = \frac{\tau}{u_c^2} \]
  where $u_c$ is the “prior” NUBBLE bottom velocity.

The subsequent iteration of NUBBLE/BBLM Cd values generally converged to within 1% of the final value after several additional time steps of model calculation (Figure 4).

Figure 4. The bottom drag coefficient Cd iteration time series associated with a trio of wave forcing conditions.

First in 99m of water, we tested the BBLM-derived bottom drag coefficient sensitivity to different amplitude ($A = 0.5, 1, 2, 5, 10,$ and 20m) 12-second surface waves. The wave information was converted using linear wave theory into the corresponding maximum bottom wave velocity and maximum bottom excursion amplitude using linear wave theory.
The coupled NUBBLE/BBLM model results (Table 2) show that 2m, 12s waves that interact with the imposed geostrophic current create stress conditions that are almost equivalent to the NUBBLE results with a constant $C_d = 0.005$ (Table 1). (Note the similarity in the total water column momentum and the velocity and kinetic energy profiles in Figures 3 & 5 respectively).

Consistent with this result, the 0.5m and 1m wave cases (5m, 10m & 20m wave cases) had higher (lower) total water column momentum than the $C_d = 0.005$ case. Table 2 results show that decreased total water column momentum is consistent with the larger shear stresses associated with larger wave amplitudes. Table 2 results also show less (more) turbulent kinetic energy ($Q_2$) in the 0.5m and 1m wave cases (5m, 10m 20m waves cases) than in the constant $C_d = 0.005$ case.

<table>
<thead>
<tr>
<th>Wave Amp (m)</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
<th>5.0</th>
<th>10.0</th>
<th>20.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-momentum</td>
<td>30391</td>
<td>30312</td>
<td>30138</td>
<td>29666</td>
<td>29239</td>
<td>28515</td>
</tr>
<tr>
<td>Y-momentum</td>
<td>809</td>
<td>1128</td>
<td>1708</td>
<td>2889</td>
<td>3732</td>
<td>4828</td>
</tr>
<tr>
<td>Total momentum</td>
<td>30402</td>
<td>30333</td>
<td>30186</td>
<td>29806</td>
<td>29476</td>
<td>28920</td>
</tr>
<tr>
<td>X-Bottom Stress</td>
<td>0.08</td>
<td>0.11</td>
<td>0.17</td>
<td>0.28</td>
<td>0.36</td>
<td>0.002</td>
</tr>
<tr>
<td>Y-Bottom Stress</td>
<td>0.002</td>
<td>0.004</td>
<td>0.009</td>
<td>0.024</td>
<td>0.04</td>
<td>0.0002</td>
</tr>
<tr>
<td>Total Bottom Stress</td>
<td>0.08</td>
<td>0.11</td>
<td>0.17</td>
<td>0.28</td>
<td>0.36</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Table 2. Total 99m water column momentum (kgm/s) and bottom stresses (kg/ms\(^2\)) associated with 12 second surface waves with different wave amplitudes.

These 99m model results are summarized in the left panels of Figure 5. Note in particular how increased surface wave amplitude leads to slightly increased eastward and northward surface current, the NUBBLE bottom current $U_b$ decreases from about 0.20 m/s for the 1m surface wave case to zero for the 20m surface wave case (also see Figure 5).

[ Yalin – Check with Chris Naimie who said that we might have doing this bottom stress calculation incorrectly.]

Figure 5 (III.2.2). The 99m depth NUBBLE results for the different (left) 12s surface wave amplitude cases and (right) constant bottom drag coefficient $C_d$ cases.

The hodographs in the left panel of Figure 6 generally show fully developed classical bottom Ekman layers (see Appendix B). With the exception of the 20m wave amplitude case, the bottom current is non-zero because of the slip bottom condition. The Ekman layer depth scales $D$ corresponding to different wave amplitudes and water depths are presented in Table 3.

Figure 6 (III.2.3). Water depth-dependent families of NUBBLE velocity results for different 12s surface waves. Because we use a slip bottom boundary condition for the model calculation, the model bottom current velocities are not necessarily zero.
C. Coupled NUBBLE / BBLM Results: Water depth Sensitivity

In this set of experiments, we tested the sensitivity of the NUBBLE / BBLM results under the same range of surface wave conditions for the additional water depths of 80m, 40m and 12m respectively. The dependence of surface and bottom velocities on different surface wave amplitudes for each of the water depths is presented in Figure 6. The family of velocity and turbulent kinetic energy profiles in a water depth of 40m under different wave amplitude is plotted in Figure 7.

Figure 7 (III.2.4). The 40m depth NUBBLE results for different 12s surface wave amplitudes.

The results in Figure 5 imply that for water depths less than 100m the presence of 2m amplitude 12s surface waves create larger bottom stresses than that for the constant Cd = 0.005 model run in 100m. This is suggested by lower surface and bottom velocity values for the different wave cases.

The depth-dependent families of velocity hodographs in Figure 6 reflect the differing frictional environments induced by the different surface wave amplitudes. In water depths greater than 80m, the hodographs vary from ones that look like classical bottom Ekman layer spirals for the smaller wave amplitudes (relatively less total bottom stress) to less complete Ekman layer spirals for the larger wave amplitudes (relatively more total bottom stress). In water depths less than 80m all of the hodographs define incomplete Ekman layer spirals for all of the wave amplitudes considered.

These velocity structures can be explained by considering the classical Ekman bottom boundary layer structure. Specifically the total Ekman layer solution for eastward geostrophic flow forcing \( U_g \), with a no-slip bottom condition according to Kundu (1990) is

\[
\begin{align*}
\mathbf{u} &= U_g \left(1 - e^{-\xi} \cos \xi\right) \\
\mathbf{v} &= U_g e^{-\xi} \sin \xi,
\end{align*}
\]

where \( \xi = \frac{h + z}{D} \) is the non-dimensional height above the bottom and \( D \) – the Ekman layer depth scale - is defined as
\[ D = \left[ \frac{2\mu}{f} \right]^\frac{1}{2}, \quad (8) \]

where \( \mu \) is depth independent eddy viscosity and \( f \) is the Coriolis coefficient. At the bottom (\( z = -h; \ \xi = 0 \)) \( u = 0 \) and \( v = 0 \). For \( \xi \to \infty \) (i.e. \( \xi \) gets very large compared to the Ekman depth \( D \), i.e. \( \xi = \pi \)) \( u = U_s \) and \( v = 0 \). Thus a complete bottom Ekman boundary layer spiral exists as long as the water depth is greater than \( \pi \times D \). The latter result and the Ekman depth scales the Table 3 help to explain why the incomplete bottom Ekman spirals are found at the various surface wave amplitudes and water depths presented in Figure 6.

Interestingly it was found that the bottom current can go to zero under some circumstances—even though we used a NUBBLE slip bottom boundary condition. For example, a 1m amplitude, 12s wave in 12m deep water will create bottom stress conditions that will cause the current velocity will go to zero at the bottom.

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Wave/Current Cd</th>
<th>Cons. Cd</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A = 0.5m</td>
<td>A = 1.0m</td>
</tr>
<tr>
<td>( \mu ) m/s</td>
<td>D (m)</td>
<td>( \mu ) m/s</td>
</tr>
<tr>
<td>99</td>
<td>0.024</td>
<td>18.22</td>
</tr>
<tr>
<td>80</td>
<td>0.028</td>
<td>19.55</td>
</tr>
<tr>
<td>40</td>
<td>0.028</td>
<td>19.63</td>
</tr>
<tr>
<td>12</td>
<td>0.011</td>
<td>12.28</td>
</tr>
</tbody>
</table>

Table 3. Vertically-averaged eddy viscosity \( \mu \) and corresponding bottom Ekman layer depth scale \( D \) for different amplitude 12 second wave and constant \( Cd = 0.005 \) cases in different water depths. The shaded boxes indicate those cases in which the bottom Ekman layer are not complete in the water column.

The dependencies of the surface and bottom velocity components on surface wave amplitudes (and thus frictional levels) in different water depths are summarized in Figure 6. Specifically for water depth deeper than about 60m, the eastward surface current \( U_s \) increases with increased wave amplitude; and the northward surface current \( V_s \) increases with increased wave amplitude for water depth deeper than about 20m. Conversely, for depths less than 60m both eastward and northward current velocities for all depths are less with increased wave amplitude (Figures 6 & 7).

We also explored the variation of drag coefficient \( Cd \) versus wave amplitude \( A \) diagram for different water depths (Figure 8). We found that in shallower depths...
(12m), the slope of Cd versus A curve is larger. In deep water (99m), the increase of Cd with increased wave amplitude is very small. As the water depth becomes shallower, this increase becomes more and more obvious. Thus, in 12m deep water, a slight increase of wave amplitude can result in a large Cd increase. This Cd versus A diagram clearly shows us that waves are very important to properly determine bottom stresses in shallower water ($H \leq 80m$). For example, the 2m amplitude wave only generates an equivalent Cd of 0.005 in 99m deep water, compared with an equivalent Cd of 0.4 in 12m deep water. Thus to generate an equivalent Cd of 0.005, we need to input a 2m, 1.2m, 0.4m, and 0.1m amplitude wave for 99m, 80m, 40m and 12m deep water separately. We also can see that in all depth the Cds versus A-dependence converges to 0.001 when there are no waves.

**Figure 8** (III.2.5.). NUBBLE bottom drag coefficient for different 12s surface wave amplitudes

### 5. Discussion

In order to have the surface current velocity increase with increased wave amplitude, the $u$ and $v$ values need to increase with decreased $\xi$ values at the surface (as discussed above, $D$ increases with increased wave amplitude, and because $\xi = \frac{h}{D}$ at the surface, $\xi$ decreases with increased wave amplitude). In that case, when $\frac{du}{d\xi} < 0$, or $\frac{3\pi}{4} < \xi < \frac{7\pi}{4}$, $U_s$ increases with increased wave amplitude. In the same way, $V_s$ increases with increased wave amplitude when $\frac{\pi}{4} < \xi < \frac{5\pi}{4}$. This means the current profile will be rearranged with the changing of bottom stress because of the influence of the bottom Ekman layer. Thus the surface current velocity actually increases with increased bottom stress. On the other hand, if the water depth is too shallow, then friction will dominant the whole water column, and surface current velocity decreases with increased bottom stress. Finally, if the water depth is too large, then the bottom Ekman layer influence does not reach the surface.

### 6. Summary of Conclusions

The dependence of combined wave/current-induced drag coefficient Cd on wave amplitude is more obvious and important in water depths $\leq 80m$. Specifically the combination of a steady imposed current and 12s surface waves, with amplitudes of 2m, 1.2m, 0.4m, and 0.1m in 99m, 80m, 40m and 12m deep water, respectively, generate a Cd of about 0.005. Waves with amplitudes larger (smaller) than those above will produce Cds $>0.005 (<0.005)$. The higher
amplitude waves cause correspondingly larger bottom shear stresses and thus more energy consumption in the water column. For depths greater than $\pi D$, the bottom Ekman dynamics-induced current rotation to the left is almost complete. However, for depths less than $D$ (which increases as wave amplitude increases), the Ekman layer current rotation is incomplete. This is most evident for depths less than 80m and typical waves. Although the water column-averaged model current is reduced under this circumstance, the model eastward (northward) surface current actually increases when the ratio of water depth ($h$) to bottom Ekman layer scale ($D$) is in the range of $\frac{3\pi}{4} < \frac{h}{D} < \frac{7\pi}{4}$; $(\frac{\pi}{4} < \frac{h}{D} < \frac{5\pi}{4})$.

7. Appendices

Appendix A. NUBBLE Governing Equations

The governing equations for NUBBLE are the linearized shallow water equations which for momentum are

$$\begin{align*}
\frac{\partial u}{\partial t} - f v &= - g \frac{\partial \eta}{\partial x} + \frac{\partial}{\partial z} \left( \mu \frac{\partial u}{\partial z} \right) \\
\frac{\partial v}{\partial t} + f u &= - g \frac{\partial \eta}{\partial y} + \frac{\partial}{\partial z} \left( \mu \frac{\partial v}{\partial z} \right)
\end{align*}$$

(A.1) (A.2)

where $u, v$ are x, y direction current and $f, \eta, g, \mu$ are the Coriolis parameter, surface elevation, gravity, and eddy viscosity respectively.

The conservation equation for energy is

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left( \kappa_h \frac{\partial T}{\partial z} \right)$$

(A.3)

where $T$ is temperature and $\kappa_h$ is heat diffusivity.

The Constitutive Relationship for Perturbation Density is

$$\rho \equiv -\alpha (T - T_0)$$

(A.4)
The use of (A.3) enables (A.4) to be written as a conservation equation for the perturbation density:

\[ \frac{\partial \rho}{\partial t} = \frac{\partial}{\partial z} \left( \kappa_h \frac{\partial \rho}{\partial z} \right) . \]  

(A.5)

The free surface boundary conditions are given in terms of the applied wind stress \( \tau_w \) and heat flux \( H \), so that

\[ \mu \frac{\partial v}{\partial z} = \frac{\tau_w}{\rho_0} \]  

(A.6)

\[ \kappa_h \frac{\partial T}{\partial z} = H \Rightarrow \kappa_h \frac{\partial \rho}{\partial z} = -\alpha \dot{H} . \]  

(A.7)

The bottom boundary conditions are given in terms of bottom stress only (heat and salinity fluxes are assumed to be negligible) according to

\[ \mu \frac{\partial V}{\partial z} = \frac{\tau_b}{\rho_0} = C_d |V_b| V_b \]  

(A.8)

\[ \kappa_h \frac{\partial T}{\partial z} = 0 \Rightarrow \kappa_h \frac{\partial \rho}{\partial z} = 0 , \]  

(A.9)

where \( C_d \) is constant drag coefficient, and \( V_b \) is the bottom velocity.

**Appendix B. Surface Current and Depth Relationship**

From eq. (6) (7) in the main text, we have:

\[ u = U_s \left( 1 - e^{-\xi} \cos \xi \right) ; \quad v = U_s e^{-\xi} \sin \xi ; \quad \xi = \frac{h + z}{D} \]

As we discussed above, \( D \) increases with increased wave amplitude, thus, \( \xi \) decreases with increase wave amplitude. So, in order to have the surface current velocity increase with increased wave amplitude, \( u \) and \( v \) values need to increase with decreased \( \xi \) values at the surface.

So, we have:

\[ \frac{\partial u}{\partial \xi} < 0 \]
That is:  
\[
\frac{\partial}{\partial \xi} \left[ U_g \left(1 - e^{-\xi} \cos \xi \right) \right] < 0
\]

\[
U_g e^{-\xi} \left( \cos \xi + \sin \xi \right) < 0
\]

\[
2U_g e^{-\xi} \sin \frac{\pi}{4} \cos \left( \frac{\pi}{4} - \xi \right) < 0
\]

since:  
\[
2U_g e^{-\xi} \sin \frac{\pi}{4} > 0
\]

so,  
\[
\cos \left( \frac{\pi}{4} - \xi \right) < 0
\]

\[
\cos \left( \frac{\xi - \pi}{4} \right) < 0
\]

\[
\frac{\pi}{2} < \xi - \frac{\pi}{4} < \frac{3\pi}{2}
\]

\[
\frac{3\pi}{4} < \xi < \frac{7\pi}{4}
\]

So, as \(\xi\) decreases from \(\frac{7\pi}{4}\) to \(\frac{3\pi}{4}\), \(u\) increases.

Same to \(v\), we have:  
\[
\frac{\partial}{\partial \xi} \left( U_g e^{-\xi} \sin \xi \right) < 0
\]

That is:  
\[
\frac{\partial}{\partial \xi} \left( U_g e^{-\xi} \sin \xi \right) < 0
\]

\[
U_g e^{-\xi} \left( \cos \xi - \sin \xi \right) < 0
\]

\[
2U_g e^{-\xi} \cos \frac{\pi}{4} \sin \left( \frac{\pi}{4} - \xi \right) < 0
\]

since:  
\[
2U_g e^{-\xi} \cos \frac{\pi}{4} > 0
\]

so:  
\[
\sin \left( \frac{\pi}{4} - \xi \right) < 0
\]

\[
\sin \left( \frac{\xi - \pi}{4} \right) > 0
\]

\[
0 < \xi - \frac{\pi}{4} < \pi
\]

\[
\frac{\pi}{4} < \xi < \frac{5\pi}{4}
\]

As \(\xi\) decreases from \(\frac{5\pi}{4}\) to \(\frac{\pi}{4}\), \(v\) increases.
Because when $\xi = \pi$, $u = U_g$, $v = 0$. So, when $\frac{3\pi}{4} < \xi < \pi$, $u > U_g$ and $v > 0$.

At surface, $z = 0$, so $\xi = \frac{h}{D}$, that is for $\frac{3\pi}{4} < \frac{h}{D} < \pi$, $u > U_g$.

8. Acknowledgments

This research benefitted greatly form the assistance provided by Chris Naimie who resides in Daniel Lynch’s Numerical Methods Laboratory in the Thayer School at Dartmouth College. The New Hampshire/Maine Sea Grant Program supported this work through grant number R/CE-122.

9. References

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Figure 1(III.1.1) The vertical discretization of the NUBBLE grid.
Figure 2. Bottom boundary layer model geometry for the Styles (1998) model, which has three boundary layer regions; one for combined wave and current $0 \leq z \leq z_w$; one for the transition layer $z_w \leq z \leq z_c$; and one for current only $z > z_c$. The horizontal currents $u_1$ and $u_2$ at elevations $z_1$ and $z_2$ respectively are used to define the near-bottom stress.
Figure 3 (III.2.1). NUBBLE results for the different constant bottom drag coefficient cases. Q2 is twice the total kinetic energy; ENZM the vertical eddy viscosity (m²/s); ENZH the vertical heat diffusivity (m²/s); ENZO the vertical diffusivity for turbulence quantities (m²/s);
Figure 4. The bottom drag coefficient $C_d$ iteration time series associated with a trio of wave forcing conditions.
Figure 5 (III.2.2). The 99m depth NUBBLE results for the different (left) 12s surface wave amplitude cases and (right) constant bottom drag coefficient Cd cases.
Figure 6 (III.2.3). Water depth-dependent families of NUBBLE velocity results for different 12s surface waves. Because we use a slip bottom boundary condition for the model calculation, the model bottom current velocities are not necessarily zero.
Figure 7 (III.2.4). The 40m depth NUBBLE results for different 12s surface wave amplitudes.
Figure 8 (III.2.5.). NUBBLE bottom drag coefficients for different 12s surface wave amplitudes.