Laboratory studies of lateral dispersion by the relaxation of diapycnal mixing events

GRANT A. STUART†, MILES A. SUNDERMEYER
AND DAVE HEBERT

1Department of Estuarine and Ocean Sciences, Univ. Mass. Dartmouth, 706 S. Rodney French Blvd., New Bedford, MA 02744, USA
2Graduate School of Oceanography, Univ. Rhode Island, 215 South Ferry Rd., Narragansett, RI 02882, USA

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Laboratory experiments in a rotating, continuously stratified fluid were used to examine the problem of lateral dispersion by the relaxation of diapycnal mixing events. Localized mixing was imposed mechanically to create isolated mixed patches, which were subsequently allowed to adjust under the influence of rotation to spin up balanced eddies. The effect of the superposition of many such eddies on lateral dispersion was quantified via the rate of spreading of a passive tracer released into the flow. Results indicate that the rate of lateral dispersion is approximately inversely proportional to the time between mixing events, and that it depends strongly on both rotation rate and buoyancy frequency of the ambient fluid through the Rossby radius of deformation and the Burger number. The importance of these parameters is consistent with theoretical predictions and numerical simulations of lateral dispersion for the same mechanism as described by previous investigators. However, the particular dependence found in the present laboratory experiments for \( Bu > 1 \) suggests a modified scaling that takes into account the finite scale of the initial mixed patches.

Key Words: Rotating tank experiments, geostrophic adjustment, diapycnal mixing, lateral dispersion

1. Introduction

1.1. Motivation and Background

The problem of lateral dispersion at sub-mesoscales has been the subject of increasing interest in recent years, as numerical models continue to push the limits of grid resolution, and as the importance of processes acting at these scales has become increasingly apparent. Direct observations of lateral dispersion using passive tracers in both coastal and open ocean settings over the past decade (e.g., Ledwell et al. 1998; Sundermeyer & Ledwell 2001; Chant et al. 2008; Dale et al. 2006) have raised numerous questions about dispersion at these scales, both in terms of the underlying mechanisms, and in terms of the basic nature of the dispersion.

In recent years, a number of theories have been put forth to explain submesoscale lateral dispersion, either as the direct result of mesoscale straining, or as the result of a down-scale cascade from the mesoscale to the submesoscale. For example, Smith &

† Present address: Fugro Global Environmental & Ocean Sciences., 6100 Hillcroft St., Houston, TX 77081, USA.
Ferrari (2009) suggest that lateral dispersion at the submesoscale can be explained solely by the forward cascade of enstrophy associated with mesoscale straining, which leads to an increase in tracer variance at small scales. Thomas et al. (2008) suggest ageostrophic motions resulting from the direct forward cascade of energy from the mesoscale to sub-mesoscale may also be significant.

Other submesoscale lateral dispersion mechanisms do not directly involve mesoscale processes at all. For example, internal wave shear dispersion as envisioned by Young et al. (1982), relies on the interaction between internal wave shear, and diapycnal diffusivity. While the latter has long been considered an important mechanism in the ocean on scales of order 1 km, however, analysis of some of the above mentioned tracer experiments suggest this cannot be the full story (e.g., Ledwell et al. 1998; Sundermeyer & Ledwell 2001) As an alternative, Polzin & Ferrari (2004) in the context of the open ocean, and Sundermeyer et al. (2005) in the context of the coastal ocean, suggest that dispersion at scales of order 1-10 km may be driven by stirring by small-scale vortices caused by patchy diapycnal mixing, or vortical mode stirring. It is this mechanism that is of primary interest to the present study.

1.2. Overview of geostrophic adjustment and vortical mode stirring

In this paper we present laboratory experiments of lateral stirring by the relaxation of diapycnal mixing events. The experiments are directly motivated by the observations and simple conceptual model of Sundermeyer et al. (2005), and the follow-on numerical experiments of Lelong & Sundermeyer (2005) and Sundermeyer & Lelong (2005). Presuming that diapycnal mixing in the ocean is patchy, following Sundermeyer et al. (2005), the basic premise is that such patches undergo a relaxation and adjustment, which in the presence of rotation, result in locally geostrophically balanced flows - also sometimes referred to as submesoscale eddies, or vortical modes. Considering the superposition of such motions as a random walk, with step size, \( S \), given by the ensuing velocity, \( U \), acting over an inertial time scale, \( 1/f \); and assuming a frequency of taking a step given by the frequency of diapycnal mixing events themselves, \( \phi \); the resultant lateral dispersion can be expressed simply as

\[
\kappa_H = \frac{1}{2} S^2 \phi \approx \frac{1}{2} \frac{U^2}{f^2} \phi. \tag{1.1}
\]

Assuming that the relevant velocity, \( U \), is given by an appropriate geostrophic velocity,

\[
U = \frac{h^2 \Delta N^2}{L f}, \tag{1.2}
\]

where \( h \) and \( L \) are the vertical and horizontal length scales of the mixed patch, respectively, and \( \Delta N \) is the difference in stratification between the mixed patch and the stratified background (i.e. \( \Delta N = N - N_{\text{patch}} \)), and allowing for the ultimate frictional decay of the stirring motions involved, Sundermeyer et al. (2005) arrived at a simple parameterization for lateral dispersion due to this mechanism,

\[
\kappa_H = \frac{1}{2} \left( \frac{h^2 \Delta N^2}{L f^2} \right)^2 \left( \frac{h^2 / \nu_B}{1/f} \right) \phi. \tag{1.3}
\]

Numerical simulations by Sundermeyer & Lelong (2005) aimed at testing the above parameterization showed good agreement with (1.3) for Burger number, \( Bu = R^2 / L^2 \sim 1 \), where \( R = \Delta N h / f \) is the relevant Rossby radius of deformation. Specifically, they found that the ‘observed’ \( \kappa_H \) in the model was consistent with (1.3) to within a constant scale factor for simulations in which \( \phi, N, h, f, \) and \( L \) were varied separately or in concert by
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more than an order of magnitude. From their simulations, Sundermeyer & Lelong (2005) also identified two regimes of eddy interaction, a “weakly non-linear” geostrophic regime, and a “strongly non-linear” turbulent regime. Although the eddy-eddy interactions characteristic of the strongly non-linear regime may contribute significantly to the overall stirring caused by the vortical mode, the details of this regime have not yet been explored, and will not be addressed here. Rather, the present study focuses on the “weakly non-linear” regime where each diapycnal mixing event and its associated motions decay before the next event occurs.

To better understand the details of the adjustment and subsequent displacement associated with individual mixing events, as a pre-cursor to the full random walk / eddy stirring simulations, Lelong & Sundermeyer (2005) conducted numerical simulations of the adjustment of a single isolated mixed patch for Burger numbers ranging from $Bu \ll 1$ to $Bu \gg 1$. Consistent with analytical results by numerous previous investigators (e.g., Ou 1986; McWilliams 1988) they found a strong dependence on $Bu$ of the adjusted velocity, the ratio of balanced kinetic to initial potential energy (a measure of how much of the initial potential energy is converted to balanced kinetic energy), and the energy ratio (a measure of the relative strength of the radiated wave field compared to balanced motions). Of particular interest with regard to the lateral dispersion problem was also the total displacement of fluid caused by the relaxation and adjustment of isolated mixed patches.

With the latter in mind, laboratory experiments of the adjustment of isolated mixed patches, combined with a retrospective examination of published analytical solutions for the adjustment of isolated lenses was conducted by Stuart et al. (2010). A significant result of those experiments was a clear picture of the $Bu$ dependence of both the radial displacement and associated adjustment velocity resulting from the relaxation of an isolated lens of well-mixed fluid in a continuously stratified rotating fluid. Of particular interest here are the results for large $Bu$, which suggest that a modified lateral diffusivity scaling based on (1.1), but significantly different from (1.3) for $Bu \gg 1$, may be appropriate.

1.3. Scope and Outline

With the above context in mind, the present study revisits the problem of lateral dispersion by the relaxation of diapycnal mixing events, and the geostrophic random walk scaling given by (1.1) - (1.3). In particular, we re-examine the adjustment and ensuing dispersion for large $Bu$, as this regime has been shown by Stuart et al. (2010) to yield balanced velocities that deviate significantly from those predicted by geostrophic scaling. As we shall show below, the question ultimately comes down to what to use for the velocity, $U$, in (1.1). For $Bu \ll 1$, (1.2) is appropriate. However, for $Bu \geq 1$, a more robust estimate of the ultimate balanced velocity is required.

As noted above, laboratory experiments by Stuart et al. (2010) examined the $Bu$ dependence of both the radial displacement and balanced velocity resulting from the adjustment of an isolated lens in a continuously stratified rotating fluid. In the present study, we now examine the effect of multiple vortices generated in this manner in terms of their effect on lateral dispersion. In section 2, the experimental set-up and laboratory measurement techniques are described. In Section 3, we present the results for a range of experiments designed to test the dependence of effective lateral diffusivity, $\kappa_H$, on the frequency of mixing events, $\phi$, and on $Bu$ through variations in the buoyancy frequency, $N$, and Coriolis parameter, $f$. Results are discussed in Section 4, while Section 5 summarizes and concludes.
2. Data and Methods

2.1. Experimental Set-Up

Laboratory experiments of the adjustment of a single isolated lens and its dependence on Burger number were examined in some detail by Stuart et al. (2010). The experimental set-up used here is similar to the one used in those experiments. However, modifications to enable multiple mixed patches, plus a number of analysis procedures differed.

The multiple eddy experiments were conducted on the same high-precision rotating turntable located at the University of Rhode Island (URI) as the single eddy adjustment runs described by Stuart et al. (2010). The table was manufactured by Australian Scientific Instruments (Australian Scientific Instruments 2010), and housed at the Geophysical Fluid Dynamics lab of the URI Graduate School of Oceanography. The experimental tank used in all experiments was a cylindrical Plexiglas tank \( (D = 1.0 \text{ m}, H = 0.48 \text{ m}) \) mounted on the rotating table. To correct for optical distortions when viewing from the side, this cylindrical tank was enclosed by a larger square Plexiglas tank. The volume between the two tanks was filled with fluid of the same density and stratification. A single, angled flat mirror at the perimeter of the tank allowed overhead cameras to photograph both plan and side views simultaneously. A Plexiglas lid was used to eliminate surface stress effects on the fluid.

As in Stuart et al. (2010), all experiments were conducted with a linearly stratified salt fluid using the two-tank method (Fortuin, 1960). The buoyancy frequency, \( N \), of the linearly salt-stratified tank was pre-determined based on the initial densities of the two tanks. This was also verified by measuring the density of fluid samples drawn from the bottom and top of the experiment tank using a refractometer. To ensure there were no vertical temperature gradients, the salt fluid was equilibrated overnight to ambient room temperature. This also allowed it to de-gas prior to each experiment. The table rotated counter-clockwise as viewed from above, with various rotation rates set for different experiments. The depth of the fluid for all experiments was 30 cm.

Localized mixed regions of fluid were formed by vertically oscillating horizontal grids positioned at mid-depth \((z = 15 \text{ cm}, \text{Fig. 1})\). The stainless steel mesh grids were 5.08-cm diameter circular disks. Slider-crank mechanisms were used to oscillate the mixing grids through a vertical stroke distance of 3.0 cm at a frequency of approximately 1.0 Hz. The positions of the eight grid mixers used for the dispersion experiments are shown in Fig. 1b. Wireless, remote PC control via LabView software and a National Instruments Fieldpoint relay module were used to automate the start/stop of the grid mixers, and to prescribe the mixer activation sequence. The numbers beside each mixer in Fig. 1b correspond to the mixer’s order in the sequence, which was used for all dispersion experiments. Each mixer was activated for 25 seconds to ensure that each mixed patch was thoroughly mixed, i.e., \( N_{\text{patch}} = 0 \). This allowed the difference in stratification between the mixed patch and the stratified background (i.e., \( \Delta N = N - N_{\text{patch}} \)) to equal the value of the buoyancy frequency, \( N \). An example of both plan and side views of a single mixed patch, visualized using dye, is shown in Fig. 2.

Green food coloring was used as a passive tracer to track the rate of dispersion during the multiple eddy experiments. For each experiment, dye was injected once at the center mixer during the initial mixing event, and the resultant dye spreading was used to determine effective horizontal eddy diffusivities. A Canon Powershot G6 7.1 megapixel digital still camera was used to take high-resolution images of the dye and was the primary data collection device during the experiments. The evolution of the dye patch was also recorded continuously using a Panasonic MiniDV digital video camcorder.
2.2. Dye Measurement & Analysis

The parameter values for the multiple eddy base run experiment are listed in Table 1. These parameters are identical to the single eddy base run of Stuart et al. (2010) except for the additional $\phi$ parameter, which represents the frequency of mixing events. Per Sundermeyer et al. (2005), strictly defined, the frequency of mixing events, $\phi$ represents the event frequency at a particular location in the water column. However, since in the present experiments the individual grid mixers are each activated only once, here $\phi$ was instead approximated as the inverse time between individual mixing events, $1/T_{mix}$, times the ratio of the volume of an individual mixed patch vs. that of the entire tank. Assuming an ellipsoidal mixed patch of height, $h$, and radius, $L$,

$$\phi = \frac{1}{T_{mix}} \frac{4hL^2}{3(0.5m^2)(0.3m)}.$$  (2.1)
Figure 2. Example of geostrophically adjusted eddy for base case laboratory experiment. (a) Side view of density anomaly stained by green dye. The parameters $h$ and $L$ are indicated. (b) Plan view of same eddy. For scale, the diameter of the mixer is 5.08 cm and the concentric circles on the bottom of the tank are at 5 cm intervals. After Stuart et al. (2010).

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Symbol</th>
<th>Lab Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coriolis parameter</td>
<td>$f$</td>
<td>0.25 rad s$^{-1}$</td>
</tr>
<tr>
<td>Buoyancy frequency</td>
<td>$N$</td>
<td>0.859 rad s$^{-1}$</td>
</tr>
<tr>
<td>Vertical length scale (half-height)</td>
<td>$h$</td>
<td>$5.0 \times 10^{-2}$ m</td>
</tr>
<tr>
<td>Horizontal length scale (radius)</td>
<td>$L$</td>
<td>$4.0 \times 10^{-2}$ m</td>
</tr>
<tr>
<td>Geostrophic velocity scaling</td>
<td>$U$</td>
<td>0.18 m s$^{-1}$</td>
</tr>
<tr>
<td>Mixing event frequency</td>
<td>$1/T_{\text{mix}}$</td>
<td>$4.4 \times 10^{-3}$ s$^{-1}$</td>
</tr>
<tr>
<td>Burger Number</td>
<td>$Bu$</td>
<td>18</td>
</tr>
</tbody>
</table>

Table 1. List of parameter values for the base case experimental run.

For the base experiment, the time between mixing events was chosen to correspond to a mixing interval of $T_{\text{mix}} = 225$ s. This was the e-folding time determined from the velocity decay of the single eddy experiments of Stuart et al. (2010), and represents the viscous decay scale. This was chosen in order to maximize the time between mixing events while still completing all mixing events before the dispersed dye significantly impinged on the tank boundaries. More specifically, in order for the stirring to remain in the “weakly-nonlinear regime” described by Sundermeyer & Lelong (2005), each eddy should decay before the next mixing event begins. As in Stuart et al. (2010), this base experimental run is used to benchmark all subsequent experiments. Note also that both here and in Stuart et al. (2010), the value of $Bu$ for the base run was consistent with realistic oceanic values, while the value of the Ekman number, $Ek = \nu/h^2 f$, though not preserved, was
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Figure 3. Plan view time evolution photographs of the base experimental run for the multi-eddy stirring experiment ($f = 0.25 \text{ rad/s}$, $\Delta N = 0.8587 \text{ rad/s}$, $1/T_{\text{mix}} = 0.0044 \text{ s}^{-1}$). Frame one indicates the activation sequence of the mixers.

Digital photography, both video and still images, provided detailed records of the multi-eddy stirring experiments. These were used to characterize both qualitative and quantitative aspects of the injected dye evolution. Fig. 3 shows the evolution of the dye patch from approximately 0 to 10 mixing periods for the base case run, where one mixing period, $T_{\text{mix}} = 225 \text{ s}$. Stepping through the evolution of the experiment, the dye response to the stirring began when the 1st mixer generated a stable core eddy. This first eddy was stained by an initial injection of green dye at the center of the mixed patch, located maintained at much less than one so that the mixed patches adjusted geostrophically, not frictionally.
Figure 4. Examples of image processing performed on digital still images. (a) Raw digital image of dye with the area outside tank masked. (b) Isolated dye concentration calculated from the grayscale intensity of isolated dye pixels. The image background and other lab equipment in the field of view were digitally removed (see text) in order to isolate the dye concentration field. The ellipse in panel (b) corresponds to the total variance calculated using the square root of the variance along the minor ($\sigma^2_a$) and major ($\sigma^2_b$) axes.

at the center of the tank. As the 2nd eddy formed (see Fig. 3, panel 3, $t = 2.01 \ T_{mix}$), it drew a dye tendril out of the core dye patch, but did not substantially mix the dye formed by the core eddy. The 3rd event did mix a portion of the dye in the core dye patch, and the adjusted eddy drew a significant amount of the dye towards it (Fig. 3, panel 4, $t = 2.96 \ T_{mix}$). This also reinforced a slight clockwise circulation that had begun to form in the tank. Subsequent mixing events mixed the dye even further throughout the tank, particularly the 4th, 5th, and 7th mixer events, which drew more dye to the lower right quadrant of the tank as the dye filled out most of the tank area.

In addition to the qualitative information obtained from the series of digital photographs, image processing combined with analysis of tracer moments was used to quantify the growth of the dye variance for each run. After experimenting with different combinations of the RGB (red, blue, green) components of the digital images, grayscale intensity was eventually used to estimate the relative dye concentration at each location (i.e., image pixel) in the tank. In this analysis, “background” images of the tank before dye was injected were first used to identify pixels that represented either the background grid drawn on the tank bottom, or visual obstructions such as the grid mixers themselves. In subsequent dye images, these pixels were then blanked out, and interpolated over via objective analysis using dye concentrations from uncontaminated pixels. Background light intensities based on full field images of the tank when no dye was present were also subtracted from the dye images to avoid bias in dye moment calculations. The resulting grayscale intensity values served as a proxy for dye concentration, from which horizontal maps of tracer concentration, $C(x, y)$, were obtained (Fig. 4).

From cleaned, position registered dye concentration images, the first three horizontal moments, i.e., total mass ($M_{00}$), center of mass ($X_{com} = M_{10}$; $Y_{com} = M_{01}$) and variance ($\sigma^2_x$, $\sigma^2_y$) were computed. The covariance ($\sigma_{xy}$) was also computed to account for any rotation in the dye second moment (e.g., Sundermeyer & Ledwell 2001). The moments are generally defined as

$$ M_{pq} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x^p y^q C(x, y) dx dy, \quad (2.2) $$
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Figure 5. Growth of the variance of passive tracer (dye) as a function of time. The effective horizontal diffusivity, $\kappa_H$, was computed as one half of the slope fitted via linear regression. Triangles along upper axis of figure correspond to times of images shown in Fig. 3.

where the subscripts $p$ and $q$ indicate zonal and meridional directions respectively. To allow for a rotating dye patch, the total variance was calculated from the variances along the minor ($\sigma_a^2$) and major ($\sigma_b^2$) axes of a Gaussian ellipse (Fig. 4) as

$$
\sigma_a^2 = \frac{1}{2} \left[ \sigma_x^2 + \sigma_y^2 + \frac{\sigma_{xy}}{\cos(\phi) \sin(\phi)} \right],
$$

$$
\sigma_b^2 = \frac{1}{2} \left[ \sigma_x^2 + \sigma_y^2 - \frac{\sigma_{xy}}{\cos(\phi) \sin(\phi)} \right],
$$

$$
\phi = \frac{1}{2} \arctan \left( \frac{2\sigma_{xy}}{\sigma_x^2 - \sigma_y^2} \right),
$$

where $\theta$ is the ellipse’s angle of rotation. The total dye variance for each digital image was estimated by

$$
\sigma^2 = \pi \sigma_a \sigma_b,
$$

which represents the area of an ellipse circumscribing the dye. The effective horizontal eddy diffusivity, $\kappa_H$, is proportional to the time rate of change of the total variance, and is given by

$$
\kappa_H = \frac{1}{2} \frac{d\sigma^2}{dt}.
$$

The growth of the total variance as a function of time was plotted for each run, and a linear slope fitted and used to estimate the effective horizontal diffusivity. Fig. 5 shows an example of the growth of the total variance for the base run experiment. Uncertainty estimates for the diffusivity calculations were determined by considering two sources of error. First, the uncertainty (at 95% confidence) associated with the linear fit of the variance was considered. These estimates gave relative diffusivity uncertainties of order 10% - 15%. Second, errors associated with inferring dye concentration from grayscale photographic images were considered. Such concentration estimates can be shown to saturate at high dye concentrations, and to miss dye at very low concentrations. To estimate the magnitude of the latter uncertainty, identical moment calculations were done for both the continuous concentration estimates as determined above, as well as
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Table 2. Summary of parameter values used for multi-eddy stirring experiments. Dash marks indicate base case parameter value was used in experiment.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$1/T_{mix}$ (s$^{-1}$)</th>
<th>$f$ (rad s$^{-1}$)</th>
<th>$N$ (s$^{-1}$)</th>
<th>$h$ (m)</th>
<th>$L$ (m)</th>
<th>$R/L$</th>
<th>$R$ (m)</th>
<th>$\kappa_H$ (m$^2$ s$^{-1}$)</th>
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<tbody>
<tr>
<td>Base Run</td>
<td>0.0044</td>
<td>0.250</td>
<td>0.859</td>
<td>0.05</td>
<td>0.04</td>
<td>4.3</td>
<td>0.172</td>
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<td>$T_{mix}$</td>
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<td>–</td>
<td>–</td>
<td>–</td>
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<td>–</td>
<td>–</td>
<td>$2.6 \times 10^{-5}$</td>
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<td>–</td>
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<td>–</td>
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<td>–</td>
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<td>0.0400</td>
<td>–</td>
<td>–</td>
<td>–</td>
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<td>–</td>
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<td>2.1</td>
<td>0.086</td>
<td>1.7</td>
<td>$1.8 \times 10^{-5}$</td>
</tr>
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3. Comparison with Theoretical Parameter Dependence

To investigate horizontal dispersion caused by vortical mode stirring, two sets of experiments were conducted. These experiments were designed to test the parameter dependence of the theoretical diffusivity scaling of Sundermeyer et al. (2005), as well as a modified diffusivity prediction that incorporates McWilliams (1988) semi-analytical velocity solution for large $Bu$ (e.g., see Stuart et al. 2010). The former uses equations (1.1) - (1.2) to estimate the effective horizontal diffusivity due to vortical mode stirring, while the latter replaces the velocity in (1.2) with the semi-analytical velocity solution of McWilliams (1988) for the adjustment of an axi-symmetric isolated lens in a continuously stratified fluid.

The first suite of experiments varied the event frequency, $\phi$, (via $1/T_{mix}$) in order to test the linear dependence on this parameter. The second suite of experiments was similar to the single eddy experiments of Stuart et al. (2010) in that the initial mixed patch parameters of $N$ and $f$ were systematically varied. However, in this case we tested the parameter dependence of $\kappa_H$ on $Bu$. The initial parameters and the observed horizontal diffusivities for each of the different $\phi$ and $Bu$ experiments are listed in Table 3. Experimental values for a base case experiment are also listed in the table for comparison.

3.1. Base Case Run

The effective eddy horizontal diffusivity estimated from the base case run was $\kappa_H = 2.3 \pm 0.3 \times 10^{-5}$ m$^2$s$^{-1}$. To compare this to theoretical predictions for stirring by the relaxation of diapycnal mixing events, we compare this diffusivity to two different values. The first is the direct prediction of Sundermeyer et al. (2005), where the geostrophic velocity given by (1.2) is assumed in (1.1), giving the effective diffusivity in (1.3). Here we find the laboratory result was nearly two orders of magnitude smaller than predicted by (1.3), the latter giving a value of $\kappa_H = 1.1 \times 10^{-3}$ m$^2$s$^{-1}$. Second, we compare the same base case diffusivity obtained in the lab to the prediction given by (1.1), except this
time with velocity, $U$, given by the semi-analytical result of McWilliams (1988). Here the appropriate parameter values from the base case lab run have been used in the analytical prediction of $U$. In this case, the observed lab diffusivity was an order of magnitude larger than the predicted value, i.e., $\kappa_H = 1.1 \times 10^{-6} \text{m}^2\text{s}^{-1}$. Note, however, that the McWilliams (1988) solution assumes a particular shape, which may not be the same as that realized in the laboratory; as such, some difference is to be expected.

From the base case laboratory simulation alone, it is unclear which theoretical prediction, if either, was more consistent with the lab results, since the base run showed an order magnitude or more difference from both predictions. As we shall see, however, the suite of experiments examining dependence on $Bu$ give some further insight into this question. Before we discuss those experiments, however, we first examine the simpler dependence on the frequency of mixing events. We then return to $Bu$ dependence in the following subsection.

### 3.2. $\kappa_H$ vs. $\phi$

The first set of experiments tested the linear parameter dependence of diffusivity, $\kappa_H$, on the frequency of diapycnal mixing events, $\phi$. In these experiments, each mixer was activated once according the prescribed sequence. For successive experiments, the time between mixing events was then varied. Five experimental runs were completed for the $\kappa_H$ vs. $\phi$ runs. The $\phi$ values for the experiments were varied over an order of magnitude, with $1/T_{\text{mix}}$ ranging from 0.0024 to 0.04 s$^{-1}$, as listed in Table 3.

For most of the $\phi$ runs (particularly the relatively lower mixing frequencies), the dye response to each stage of the mixing sequence was qualitatively similar to the dye evolution of the base case stirring. There was, however, a notable difference in the dye response as the frequency of mixing events increased. An example showing the difference between lower and higher $\phi$, i.e., $1/T_{\text{mix}} = 0.04 \text{ s}^{-1}$ vs. $1/T_{\text{mix}} = 0.0024 \text{ s}^{-1}$, respectively, is shown in Fig. 6. For each of the time steps, the area enclosing the dye is larger in the higher $\phi$ run than at a corresponding time in the lower $\phi$ run. It was also notable that for higher event frequency, the stirring appeared to approach the ‘nonlinear regime’ referred to by Sundermeyer & Lelong (2005), in which more than one eddy was active in the tank at any given time. However, given the relatively short time of the present experiments compared to the likely time required for nonlinear effects to become significant, it is unclear whether such effects played a part in the final diffusivity estimated from this run.

The latter point aside, qualitatively, higher $\phi$ values appeared to correspond to higher $\kappa_H$ values. This observation was confirmed quantitatively in Fig. 7, which shows the observed eddy diffusivities, listed in Table 3, plotted against the corresponding mixing frequency for each experiment. Here the observed $\kappa_H$ values clearly showed a positive linear dependence on mixing event frequency. This supports the linear scaling relationship of (1.1) associated with the random walk formulation. Note also that since varying $\phi$ does not effect $Bu$, after accounting for the linear dependence on $\phi$, the diffusivities for all $\phi$ runs were in a range consistent with the base run. Specifically, considering the full suite of $\phi$ runs, diffusivities were up to 2 orders of magnitude smaller than the Sundermeyer et al. (2005) scaling, and up to 1 order of magnitude larger than the McWilliams (1988) modified solution (Fig. 8).

### 3.3. $\kappa_H$ vs. $Bu^{1/2}$

As indicated above, the aim of the second set of experiments was to test the parameter dependence of horizontal diffusivity, $\kappa_H$, on Burger number, $Bu$. The base run of the $\phi$ experiments also served as the base run here. However, this time the value for $T_{\text{mix}}$ was held constant at 0.004 s$^{-1}$ for all $Bu$ dispersion experiments. To test the $\kappa_H$ dependence
Figure 6. Plan view photographs contrasting multi-eddy experiments of $1/T_{\text{mix}} = 0.04$ s$^{-1}$ (left column) with $1/T_{\text{mix}} = 0.0024$ s$^{-1}$ (right column) at $15 T_{\text{inertial}}$, $30 T_{\text{inertial}}$, and $60 T_{\text{inertial}}$, where $T_{\text{inertial}} = 2\pi/f = 25$ s. For the higher frequency experiment (left column), there was a larger variance at corresponding times to the lower frequency experiment (right column).

on $Bu$, both $f$ and $N$ were systematically varied in the same manner as in the single eddy experiments of Stuart et al. (2010) so as to change the value of $R$. This resulted in three $R/L$ (i.e., $Bu^{1/2}$) ratios for this suite of experiments, namely 2.1, 4.3, and 8.6 (Table 3). Calculations of the total dye variance and effective eddy diffusivity were completed using the same procedure as the $\phi$ runs.

The observed diffusivity values for the $\kappa_H$ vs. $Bu$ experiments are shown in Fig. 8 plotted against corresponding $R/L$ ratios for each experiment. The $\phi$ experiments are also included at $Bu = 4.3$ for comparison. In this figure, all diffusivities listed in Table 3
Figure 7. Observed $\kappa_H$ vs. $1/\Delta T_{mix}$, where $1/\Delta T_{mix}$ is used as a proxy for mixer frequency, $\phi$.

have been normalized by $L^2\phi/Ek$, i.e., a simple random walk with step size given by the initial scale of the anomaly, $L$, times step frequency, $\phi$, divided by the Ekman number. Two theoretical curves are also plotted for comparison with the data. Per (1.3), the Sundermeyer et al. (2005) geostrophic random walk scaling gives a normalized curve that scales as $Bu^2$, i.e., a slope of 4 on the normalized $\kappa_H$ vs. $Bu^{1/2}$ plot. Meanwhile, the normalized $\kappa_H$ curve using the McWilliams (1988) derived velocity gives the same slope of 4 for $Bu \ll 1$, but tends to 1 (i.e., the simple random walk scaling with step size $L$) for $Bu \gg 1$. Comparing the laboratory results to the theoretical predictions, we find that the lab results clearly deviate from the $(R/L)^4$ scaling for $Bu > 1$, such that for the largest Burger number experiment, $Bu^{1/2} = 8.6$, the difference is more than 3 orders of magnitude (Fig. 8). While we are mindful not to infer too much from the small range of $Bu$ explored here, the roll-off for $Bu > 1$ indicated by the modified diffusivity scaling using McWilliams (1988) velocity solution is much more suggestive of the behavior of the observed lateral dispersion than is the Sundermeyer et al. (2005) scaling for the dynamical range $1 < Bu^{1/2} < 10$ examined in the laboratory. Importantly, however, we also note that the random walk scaling using the McWilliams (1988) velocity asymptotes to the Sundermeyer et al. (2005) scaling for $Bu \ll 1$.

A final result worth noting was that when comparing runs in which both $N$ and $f$ was changed, but $R$, and thus $Bu$, was held fixed, the calculated diffusivity did not change appreciably. For example, when the base run was compared to the experiment in which both $f$ and $N$ were doubled such that $Bu$ remained fixed, the base run value, $\kappa_H = 2.3 \pm 0.3 \times 10^{-5} \text{ m}^2\text{s}^{-1}$, was quite close to $\kappa_H = 1.8 \pm 0.8 \times 10^{-5} \text{ m}^2\text{s}^{-1}$. Similarly, when $Bu$ was doubled in two separate parameter runs, the resultant diffusivity values for each run were $6.0 \pm 2.4 \times 10^{-6} \text{ m}^2\text{s}^{-1}$ and $6.1 \pm 0.4 \times 10^{-6} \text{ m}^2\text{s}^{-1}$ respectively. An analogous result was obtained for the $Bu = 2.1$ runs. These results can be seen listed in Table 3 and plotted in Fig. 8. The key point is that this finding further supports our conclusion that the modified scaling captures the $Bu$ parameter dependence correctly.
Figure 8. All observed experimental diffusivities vs. Burger number, $Bu$. All data are normalized by $\frac{1}{2}L^2 \phi / \text{Ek}$, i.e., a simple random walk with length scale $L$ and step frequency $\phi$, divided by the Ekman number. Both the $\kappa_H$ vs. $\phi$ experiments and the $\kappa_H$ vs. $Bu$ experiments are shown. The Sundermeyer et al. (2005) diffusivity scaling and the modified scaling argument incorporating the velocity solution of McWilliams (1988) are plotted for comparison.

4. Discussion

The multi-eddy stirring experiments presented here demonstrate in the laboratory how episodic diapycnal mixing in a linearly stratified water column can produce geostrophic eddies that combine to drive horizontal dispersion of a passive tracer. Besides successfully reproducing in the lab the dynamics involved in vortical mode stirring, a significant result of this study was that the observed diffusivity parameter dependence for $Bu > 1$ was more consistent with a modified theoretical diffusivity prediction using an analytical velocity solution for a continuously stratified axisymmetric lens than the simple geostrophic scaling argument of Sundermeyer et al. (2005). Observed diffusivities from the $\kappa_H$ vs. $\phi$ experiments indicated a linear relationship with changing event frequency, $\phi$ (Fig. 7). This dependence confirmed both the predicted scaling as well as basic intuition. Namely, the shorter time between mixing events allowed a larger number of eddies in a given time to act on a passive tracer. More notably, for $Bu > 1$ the observed diffusivities from the $\kappa_H$ vs. $Bu$ experiments indicated a parameter dependence more consistent with the modified theoretical diffusivity prediction incorporating the McWilliams (1988) semi-analytical velocity solution (see Eq. (1.1)). Concomitantly, also for $Bu > 1$, the observed $Bu$ dependence of effective lateral diffusivity diverged significantly from the Sundermeyer et al. (2005) diffusivity scaling.

Given the numerous approximations and constraints on the present laboratory experiments, the precise dependence of $\kappa_H$ on $Bu$ is also subject to a number of caveats. One is simply the limited range of $Bu$ that could be explored in the laboratory due to experimental constraints such as tank size, and the relative size of $N$ and $f$. On one hand, $L$ could not be increased significantly beyond the value used here, since doing so would push it too close to the tank dimensions to allow for multiple mixed patches. Alternatively, reducing $R$ could only be done by increasing $f$ or decreasing $N$, which were already quite close to one another in value; or by decreasing $h$, which would have led to
mixed patches increasingly feeling the effects of friction. The latter in particular would have in turn limited the applicability of geostrophic scaling.

A second caveat is our use of a finite number of mixing events, which might not have sufficiently approximated a statically random process; the latter being an assumption formally required by the random walk formulation. Related to this, it is worth noting that during one run in which the order of two mixers was accidentally exchanged, the dye response was altered. This suggests that the mixer sequence was, in fact, important, and therefore strictly would have violated the theoretical assumption that the mixing should be statistically random. To address both of these issues in future experiments of this type, a larger tank and increasing the number of mixing events may yield different results. In such a setup, mixing events could also be repeated in the same location after the previous eddy had dissipated away and before significant dye amounts had reached the tank boundary. This would have more closely replicated the definition of \( \phi \), which formally represents the frequency of a mixing event at a specific location.

5. Summary and Conclusions

In this study we presented laboratory experiments of lateral dispersion driven by the geostrophic adjustment of small-scale mixed regions in a continuously stratified rotating fluid. Our main result is that the sequence of turbulent diapycnal mixing, geostrophic adjustment, and linear eddy interaction yielded effective lateral diffusivities that were qualitatively and quantitatively consistent with a modified form of the random walk diffusivity prediction of Sundermeyer et al. (2005) that incorporates the semi-analytical solution for geostrophic adjustment velocity of McWilliams (1988).

The experiments presented here build on experiments by Stuart et al. (2010), which focused on the isolated adjustment of single mixing events. Their results showed that measured velocities and radial displacements depend on the Burger number, \( Bu \), given by the ratio of the Rossby radius of deformation, \( R \), to the initial horizontal scale of the mixed patch, \( L \). A key finding of Stuart et al. (2010) was that simple geostrophic velocity scaling based on initial mixed patch parameters overestimated the observed balanced velocity by 1-2 orders of magnitude for \( 1 \leq R/L \leq 10 \) (e.g., see Fig. 3 of Stuart et al. 2010), consistent with published analytical solutions for the same problem by McWilliams (1988).

Since the vortical mode stirring / random walk scaling of Sundermeyer et al. (2005) assumes geostrophic velocity scaling per (1.1) - (1.3), independent of \( Bu \), the results of Stuart et al. (2010) from the single eddy experiments suggests a modified form of the lateral diffusivity scaling of Sundermeyer et al. (2005). To this end, the present laboratory results were compared to a modified diffusivity prediction given by (1.1) in which \( U \) was taken as the analytical adjustment velocity of McWilliams (1988) continuously stratified adjustment solution. Considering both the original and the modified diffusivity parameterizations, the present multiple eddy stirring experiments were more consistent with the modified diffusivity prediction for \( Bu > 1 \).

Although the present laboratory experiments did not specifically test \( Bu \leq 1 \), the modified scaling using McWilliams (1988) solution asymptotes to the Sundermeyer et al. (2005) diffusivity scaling for small \( R/L \). This suggests that the modified theoretical diffusivity prediction using the McWilliams (1988) solution provides a more general form to predict dispersion rates from vortical mode stirring for both large and small \( Bu \). The multiple eddy experiments presented here also are consistent with the presumed positive, linear dependence of diffusivity \( \kappa_H \) on the frequency of mixing events, \( \phi \), for vortical mode stirring.
The broader impact of this study on understanding lateral stirring by vortical mode stirring is that in regions where $Bu$ is much greater than one, the applicability of simple geostrophic velocity scaling for isolated geostrophic adjustments is diminished, and the random walk parameterization of Sundermeyer et al. (2005) no longer applies. The McWilliams (1988) velocity solution suggests that for $Bu < 1$ the Sundermeyer et al. (2005) simple parameterization should still hold. Thus, using a modified random walk diffusivity scaling with $Bu$ dependent solution for adjustment velocity extends the usefulness of a modified diffusivity parameterization to ocean regions characterized by a wider range of Burger number.

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