Problem Set # 3

1. Why do we use $U_{10}$ the wind speed 10 m above the surface, in the drag formula

$$\tau = \rho C_D U_{10}^2$$

where $C_D =$

$$10^{-3} \quad U_{10} < \frac{5 \text{ m}}{\text{sec}}$$

$$2.5 \cdot 10^{-3} \quad U_{10} > \frac{5 \text{ m}}{\text{sec}}$$

Why not use the wind speed right at the surface? Explain. Why do you think that the drag coefficient $C_D$ has a large change in value when the wind speed (at 10m) increases above? Hint: what would cause a sudden increase in air turbulence with an increasing wind speed? Show that

$$\frac{u_{air}^*}{U_{10}} = \sqrt{C_D}$$

$$u_w^* = \sqrt{\frac{\rho_{air}}{\rho_w}} u_{air}^* = 3\% \text{ of } u_{air}^*$$

2. Consider a steady state, narrow channel flow whose depth is $D = 10$ m. There is no wind. The surface has a constant surface slope, $|\alpha| = 10^{-6}$ estimate the turbulent velocity at the surface, at the mid depth, and on the bottom. Where is the turbulence a maximum?

3. Suppose the turbulent eddy viscosity in problem 2 is given by $\kappa = 10^4 \frac{m^2}{\text{sec}}$. Estimate the mean velocity at the surface, at the mid depth, and on the bottom.

4. Suppose the surface in the channel of problem # 2 is completely flat, $\alpha = 0$, but there was wind with a speed of $U_{10} = 10 \frac{\text{m}}{\text{sec}}$ at a height of 10 m above the surface. Calculate the stress and estimate the turbulent velocity: (a) at the surface; (b) at mid depth; (c) on the bottom.

5. The wind blows along a coastal channel of depth 40 meters with a speed of 10 m/sec. If the drag coefficient between the water and air is found to be $2.5 \times 10^{-3}$, (a) calculate the stress induced by the wind on the water surface; (b) calculate the in air friction velocity. the in water friction velocity; (c) what is the stress on the bottom of the water? (d) what is the stress at the surface, mid depth, and .5m from the bottom? (e) At what distance from the bottom do you expect the log layer to occur? (Hint: the current shear should be continuous with depth). (f) what is the shear $\frac{\partial u}{\partial z}$ at the surface, mid depth, .5m from the bottom? Take $\kappa^*_e = 10^2 \frac{m}{\text{sec}}$. 


6. For problem 5 (a) what is the surface current? (b) What is the current at mid depth and .5 m from the bottom? Hint for the latter you need to estimate $z_0$.

7. Suppose there is a surface input of heat of $H = 200 \text{ w/m}^2$ over water of depth 20m over sufficiently long time that a steady state is set up resulting in a surface temperature of 20 °C. The thermal diffusivity is estimated as $k = 10^{-4} \text{ m}^2/\text{sec}$. (a) If there is no upwelling what is the temperature gradient and temperature at depths of $\bar{z} = D - z = 5m, 10m, 20m$? If there was an upwelling velocity of $w = .01 \text{ mm/sec}$ what is the temperature gradient and temperature at depths $\bar{z} = D - z = 5m, 10m, 20m$?