MAR 610
Spring 2009

Problem Set # 3

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1. Calculate the Fourier Transform defined by $g(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, f(t) \exp^{(-i\omega t)}$ of the following functions:

A. $f = \delta(t - t_0)$

B. $f = \exp(-\frac{|t|}{t_0})$

C. $f = \exp[-(\frac{t}{t_0})^2]$

D. $f = H(t)$

where $H(t) = 1$ for $|t| < t_0$

$H(t) = 0$ for $|t| \geq t_0$

E. $f = 1$

2. Calculate the Inverse Fourier Transform defined by $f(t) = \int_{-\infty}^{\infty} d\omega g(\omega) \exp^{(i\omega t)}$ of the following functions

A. $g = \delta(\omega - \omega_{0_0})$

B. $g = \cos(\omega t_0)$

C. $g = \sin(\omega t_0)$

D. $g = H(\omega)$

where $H(\omega) = 1$ for $|\omega| < \omega_0$

$H(\omega) = 0$ for $|\omega| \geq \omega_0$

3. Suppose a turbulent velocity field u' has the autocovariance function

$$R(\tau) = < u'^2 > e^{-\tau^2/\tau_0^2}$$

where τ_0 is a constant with units of time.

(a) What is the autocovariance function?
(b) What is the structure function?
(c) What is the normalized autospectra, i.e. Fourier transform of autocovariance function? (See answer to problem 1c.)
(d) What is its turbulent integral scale?
(e) What is the autocorrelation function and spectrum for acceleration fluctuations $\frac{\partial u'}{\partial t}$?
(f) What is its Taylor microscale?

4. Suppose a turbulent velocity u' with an rms value $< u'^2 >^{\frac{1}{2}} = 4 \cdot 10^{-4} \frac{m}{\sec}$ has the following frequency spectrum

$$\Phi_{1s}(\omega) = A \quad \text{for } \omega \le \omega_0$$
$$\Phi_{1s}(\omega) = B\omega^{-2} \quad \text{for } \omega_1 \ge \omega > \omega_0$$
$$\text{with } \omega_0 = .1 rad/\sec \text{ and } \omega_1 = 10 rad/\sec$$

Calculate A, B, the temporal integral scale and microscale, and the rms value of the acceleration $< (\frac{\partial u'}{\partial t})^2 >^{\frac{1}{2}}$