MAR 610
Spring 2009

Problem Set # 2

1. Given an incompressible 2D flow with "x'" and "z" velocity given by

$$u = \alpha x$$

$$w = \beta z$$

(a) how are α and β related?

(b) If the initial positions (t = 0) are given by X, Z what are the fluid positions x, z, in terms of X,Z, t?
(c) Sketch the x,z fluid element trajectory
(d) Express the velocity field in Lagrangian coordinates.

2 The following is a very simple model for coastal upwelling on a shelf of depth d and width L.

$$u = \frac{x}{L} U$$

$$w = -\frac{Wz}{d} \qquad -d \le z \le 0$$

where the ocean surface is at z = 0, x = 0 is taken as the "western" boundary. Note that z is negative downward.

(a) Take d = 100meters, L = 50km, W = 1mm/sec. How long does it take a fluid element located on the bottom 10 km from the coast to reach a depth of 50 meters? What is its velocity at that point. How far is it from the coast at that point?
(b) How long does it take this fluid element to reach the surface? How far from the coast is the fluid point when it reaches the surface? Explain your answer and comment on what range of values of x and z this model should be valid.

3. For a 2d pure shear flow

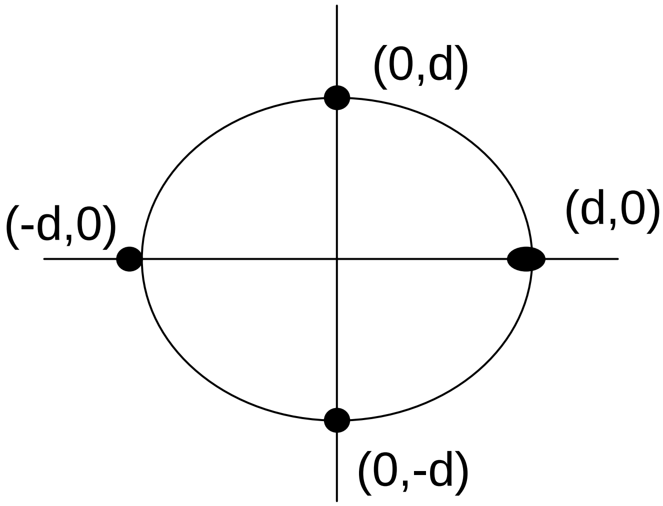
$$u = \alpha z$$

$$w = 0$$

(a) If the initial positions (t = 0) are given by X, Z what are the fluid positions x, z, in terms of X, Z, t?

(b) Consider the effect of this flow in the circle below. Using d = 1 meter, $\alpha = .02\sec^{-1}$, t = 60 seconds Show where the circle of fluid shown below evolves. How much has the circle rotated? (See book and/or notes for definition of rotation angle.)

(c) What is the direction of the rate of strain principal axis?



4. Repeat part (b) and (c) of Problem 3 for the flow field given in problem 1 for d = 1, $\alpha = .02\sec^{-1}$, t = 50 seconds.

5. Show that the stretching term $\vec{\omega}\cdot\nabla\vec{u}$ in the vorticity equation results in the magnitude of the vorticity increasing in along the direction of the principal axis of strain rate. (Ignore the viscous term which can only result in the spinning down of the vorticity.) Take the strain rate tensor along the principal axis to be

$$\tilde{S}_{ij} = \begin{matrix} \tilde{S}_{11} & 0 & 0 \\ 0 & \tilde{S}_{22} & 0 \\ 0 & 0 & \tilde{S}_{33} \end{matrix}$$

with

$\tilde{S}_{11} > 0$

$\tilde{S}_{22} = \tilde{S}_{33}$