Mar610/ MNE 490 Ocean Turbulence

Problem Set # 1

1. Prove that $\epsilon_{ijk}\epsilon_{lmk} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$. Do this both by considering components and by using

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$

2. Hence prove $\epsilon_{ijk}\epsilon_{ljk} = 2\delta_{il}$ and $\epsilon_{ijk}\epsilon_{ijk} = 6$.

3. Using suffix notation and the summation convention, prove that

$$\mathbf{u} \cdot \nabla \mathbf{u} = \boldsymbol{\omega} \times \mathbf{u} + \boldsymbol{\nabla}(\tfrac{1}{2}q^2)$$

   where $\boldsymbol{\omega} = \nabla \times \mathbf{u}$ (ω is called the vorticity) and $q = |\mathbf{u}|$.

4. Vorticity, the curl of velocity field, can also be written as

$$\omega_i = \epsilon_{ijk}\frac{\partial u_k}{\partial x_j} \tag{1}$$

   . The velocity gradient tensor $\frac{\partial u_k}{\partial x_j}$ is also called the deformation rate.

   (a) Split the deformation rate into a symmetric part and an antisymmetric part. These are called the strain rate s_{kj}, and the rotation tensor r_{kj}, i.e,

$$\frac{\partial u_k}{\partial x_j} = s_{kj} + r_{kj}. \tag{2}$$

   Write s_{kj} and r_{kj} in terms of the velocity gradients.

   (b) Using the definition of vorticity and the result from part(a), show that

$$\omega_i = \epsilon_{ijk}r_{kj} \tag{3}$$

   .

   (c) Using the result from Problem 1, and part (b), show that

$$r_{ij} = -\frac{1}{2}\epsilon_{ijk}\omega_k \tag{4}$$

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