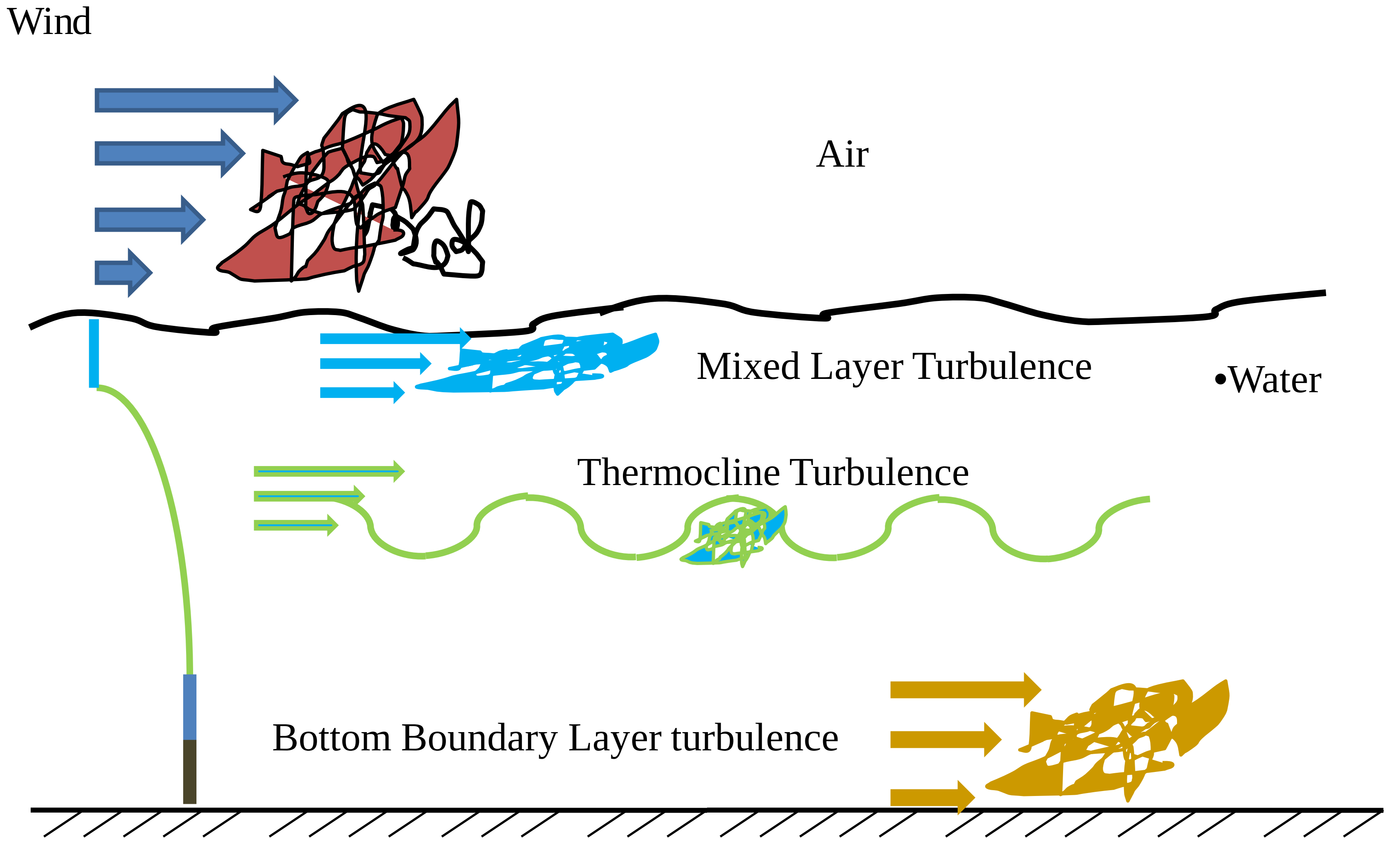
Date	Topic	Source:	HW
1/27/2009		Introduction	
1/29/2009	Math Review	Math_Review	PS_1
2/3/2009	Fluid Flow Kinematics/Conservation Laws	Class Notes	PS_2
2/5/2009			
2/10/2009	HW Session PS _1		
2/12/2009	HW Session PS _2	SpecExample& More Math Notes	
2/17/2009	No Class Follow Monday Schedule		
2/19/2009	Averaging and Spectra II	Notes_02_19	PS_3
2/24/2009			
2/26/2009	HW Session PS _3		
3/3/2009	Exam # 1	Exam#1	
3/5/2009	Mixing and dispersion	Notes-Mixing and Dispersion	PS_4
3/10/2009	Introduction to Turbulence: Mean flow Effects	Notes:3_10_12_24	PS_ 5
3/12/2009			
3/24/2009			
3/26/2009	HW Session PS4, PS_5		
3/31/2009	Ekman Dynamics	Notes:3_26_31	PS_6
4/2/2009			
4/7/2009	HW Session PS6		
4/9/2009	Review		
4/14/2009	Exam # 2		
4/16/2009	Turbulent Flucutations	Notes:4_16_4_23, TL1-2	PS_7
4/21/2009			
4/23/2009			
4/28/2009	HW Session PS 7		
4/30/2009	Turbulent Dynamical Scaling and Spectra	Notes:4_30_5_5, TL-3, 8	PS_ 8
5/5/2009			
5/7/2009	HW Sesssion PS_8		
5/12/2009	Review		
	Final Exam Date, Time, and Location TBD		

Ocean Turbulence (3D, Microstructure)



Wind

Air

Mixed Layer Turbulence

- Water

Thermocline Turbulence

Bottom Boundary Layer turbulence

How does the turbulence affect the mean flow?

Mean Flow

$$\bar{u}$$

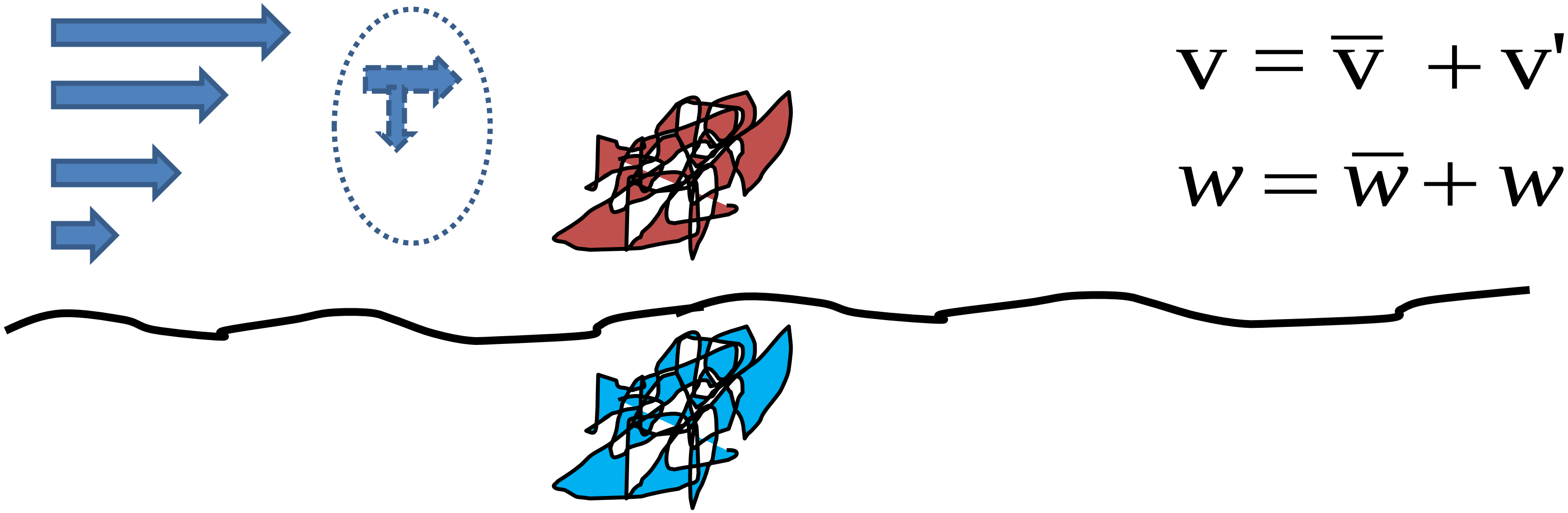
3D turbulence

$$u', v', w'$$

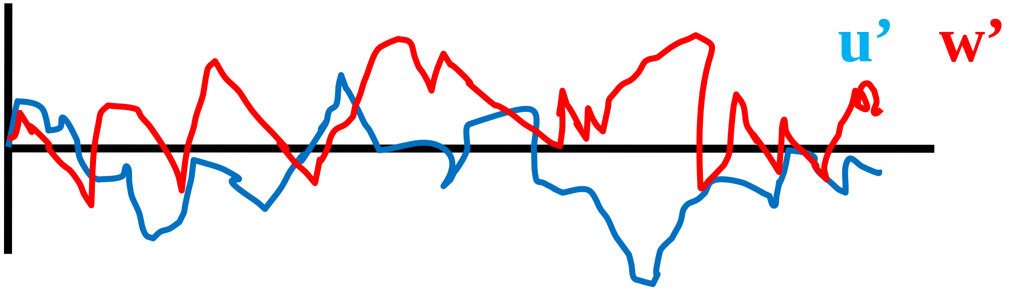
$$u = \bar{u} + u'$$

$$v = \bar{v} + v'$$

$$w = \bar{w} + w'$$



$$< u'w' > \; < 0$$



Concept of Reynolds Stress

$$\tau = -\rho < u'w' >$$

Momentum Equations with Molecular Friction

$$\frac{du}{dt} - f\mathrm{v} = -\frac{1}{\rho}(\frac{\partial p}{\partial x}) + Fr_{\mathrm{x}}$$

$$\frac{dv}{dt} + fu = -\frac{1}{\rho}(\frac{\partial p}{\partial y}) + Fr_y$$

$$\frac{dw}{dt} = -\frac{1}{\rho}(\frac{\partial p}{\partial z}) - g + Fr_z$$

where

$$\frac{d}{dt} = u\frac{\partial}{\partial x} + \mathrm{v}\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

But

$$Fr_{x,y,z} = \nu\nabla^2(\mathrm{u,v,w})$$

$$\nu = \text{molecular viscoisty} = 10^{-6}\,\frac{m^2}{\text{sec}}$$

Approach for Turbulence

$$\frac{du}{dt} - f\mathrm{v} = -\frac{1}{\rho}(\frac{\partial p}{\partial x})$$

$$\frac{dv}{dt} + fu = -\frac{1}{\rho}(\frac{\partial p}{\partial y})$$

$$\frac{dw}{dt} = -\frac{1}{\rho}(\frac{\partial p}{\partial z}) - g$$

where

$$\frac{d}{dt} = u\frac{\partial}{\partial x} + \mathrm{v}\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

$$\text{But } \mathrm{u_i} = \overline{\mathrm{u}}_{\mathrm{i}} + \mathrm{u'_i}$$

$$\mathrm{u_x} = \overline{u} + u'$$

$$\mathrm{v} = \overline{\mathrm{v}} + \mathrm{v'}$$

$$\mathrm{w} = \overline{w} + \mathrm{w'}$$

Example
Uniform unidirectional wind blowing over ocean surface

Dimensional Analysis

Boundary Layer Flow

- Gradient in “x” direction smaller than in “z” direction

Example:Mean velocity unidirectional , no gradient in “y” direction

$$\boxed{\frac{du}{dt} - f\mathrm{v} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)}$$

$$\frac{d\mathrm{v}}{dt} + fu = -\frac{1}{\rho}\left(\frac{\partial p}{\partial y}\right)$$

$$\frac{dw}{dt} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial z}\right) - g$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + \mathrm{v}\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \mathrm{v}\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - f\mathrm{v} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)$$

using $\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}\mathrm{v} - \frac{\partial}{\partial z}w = 0$

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{uu}{2}\right) + \cancel{\frac{\partial(\mathrm{v}u)}{\partial y}} - \frac{\partial(wu)}{\partial z}) - f\mathrm{v} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\partial (uw)}{\partial z} - fv = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)$$

$$\text{But } \mathrm{u_i} = \mathrm{\bar{u}_i} + \mathrm{u'_i}$$

$$u = \bar{u} + u'$$

$$\mathrm{v} = \bar{\mathrm{v}} + \mathrm{v'}$$

$$\mathrm{w} = \bar{w} + \mathrm{w'}$$

$$(\bar{u} + \cancel{u'})\frac{\partial(\bar{u} + \cancel{u}')}{\partial x} + \frac{\partial(\bar{u} + u')(\bar{w} + w')}{\partial z}$$

Now we average the momentum equation

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \frac{\partial(<u'w'>)}{\partial z} - f\bar{\mathrm{v}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x}$$

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} - f\bar{\mathrm{v}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + \frac{1}{\rho}\frac{\partial \tau_x}{\partial z}$$

where

$\tau_x = -\rho <u'w'> = "x"$ component of Reynolds Stress

General Case of Vertical Turbulent Friction

$$\frac{\partial \overline{u}_i}{\partial t} + \overline{u}_j \frac{\partial \overline{u}_i}{\partial x_i} + (\vec{f} \times \vec{\overline{u}})_i = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} + \frac{1}{\rho}\frac{\partial \tau_i}{\partial z} - g_i$$

where

$$\tau_{\mathrm{i}} = -\rho < u_i{}' w' > \quad \text{for } u'_x = u', u'_y = \mathrm{v}'$$

$$= 0 \qquad \mathrm{i} = 3\ (\mathrm{z})$$

Note that we sometimes use 1,2,3 in place x, y, z as subscripts

Convention: When we deal with typical mean equations we drop the “mean” Notation!

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} + (\vec{f} \times \vec{u})_i = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{1}{\rho}\frac{\partial \tau_i}{\partial z} - g_i$$

Component form of Equations of Motion with Turbulent Vertical Friction

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\mathrm{v}\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}-f\mathrm{v}=-\frac{1}{\rho}(\frac{\partial p}{\partial x})+\frac{1}{\rho}\frac{\partial \tau_x}{\partial z}$$

$$\frac{\partial \mathrm{v}}{dt}+u\frac{\partial \mathrm{v}}{\partial x}+\mathrm{v}\frac{\partial \mathrm{v}}{\partial y}+w\frac{\partial \mathrm{v}}{\partial z}+fu=-\frac{1}{\rho}(\frac{\partial p}{\partial y})+\frac{1}{\rho}\frac{\partial \tau_y}{\partial z}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+\mathrm{v}\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}(\frac{\partial p}{\partial z})-g$$

Note: in many cases the mean vertical velocity is small and we can assume w = 0 which leads to the hydrostatic approximation and

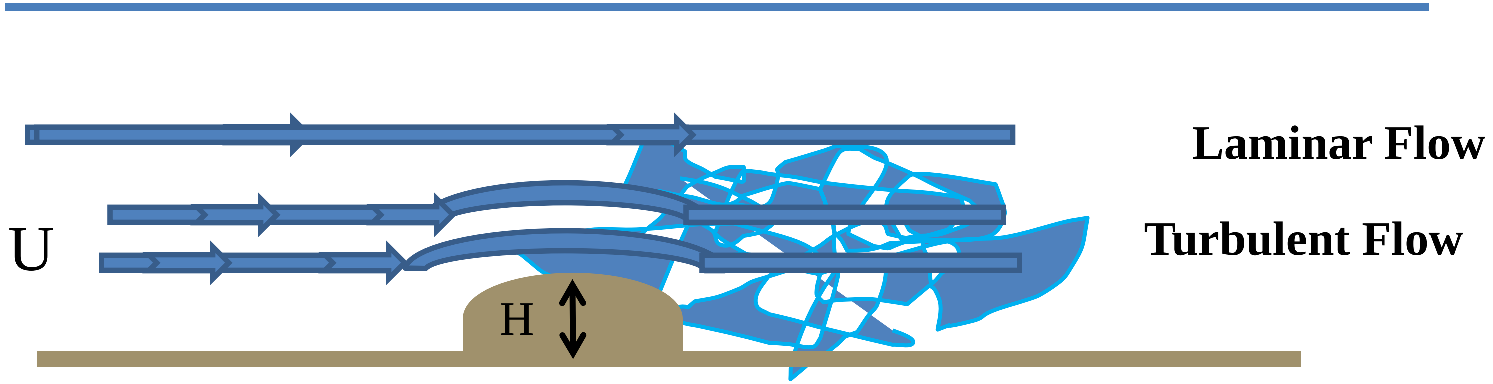
$$(1)\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\mathrm{v}\frac{\partial u}{\partial y}-f\mathrm{v}=-\frac{1}{\rho}(\frac{\partial p}{\partial x})+\frac{1}{\rho}\frac{\partial \tau_x}{\partial z}$$

$$(2)\frac{\partial \mathrm{v}}{dt}+u\frac{\partial \mathrm{v}}{\partial x}+\mathrm{v}\frac{\partial \mathrm{v}}{\partial y}+fu=-\frac{1}{\rho}(\frac{\partial p}{\partial y})+\frac{1}{\rho}\frac{\partial \tau_y}{\partial z}$$

$$(3)\quad\frac{\partial p}{\partial z}=-\rho g$$

Unstratfied flow $\rho = \rho_0$ constant

Example: Tidal flow over a mound



Reynolds Number $\mathrm{Re} = \frac{uL}{\nu}$;

u, L characteristic values of the mean flow, $\nu = 10^{-6} \frac{m^2}{\sec}$

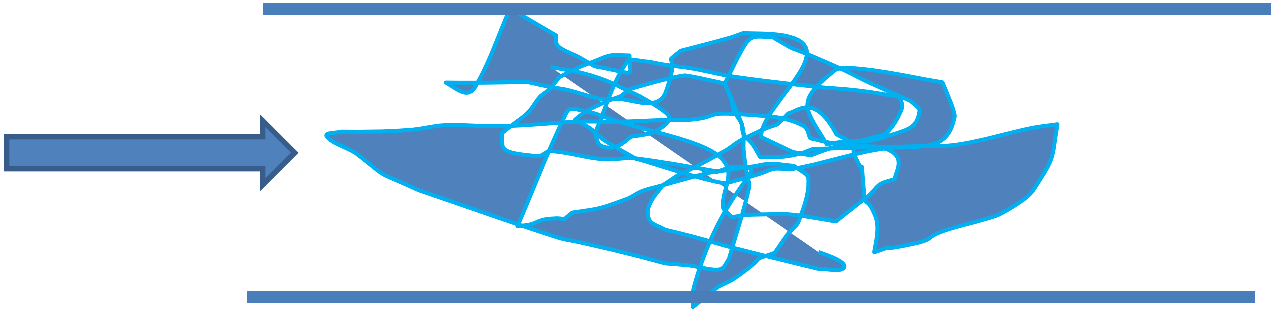
For unstratfied flow $\rho = \rho_0$ constant

Turbulence occurs when $\mathrm{Re} > \mathrm{Re}_c \sim 3000$

3 D Turbulence: Navier Stokes Equation

(no gravity, no coriolis effect)

Examples: tidal channel flow, pipe flow, river flow, bottom boundary layer)



I II III IV

$$\frac{\partial u_i}{\partial t} + \{u\frac{\partial u_i}{\partial x} + \mathrm{v}\frac{\partial u_i}{\partial y} + w\frac{\partial u_i}{\partial z}\} = -\frac{1}{\rho_0}\frac{\partial p_i}{\partial x_j} + \nu(\frac{\partial^2 u_i}{\partial x^2} + \frac{\partial^2 u_i}{\partial y^2} + \frac{\partial^2 u_i}{\partial z^2})$$

I . Acceleration
II . Advection (non-linear)
III. Dynamic Pressure
IV. Viscous Dissipation

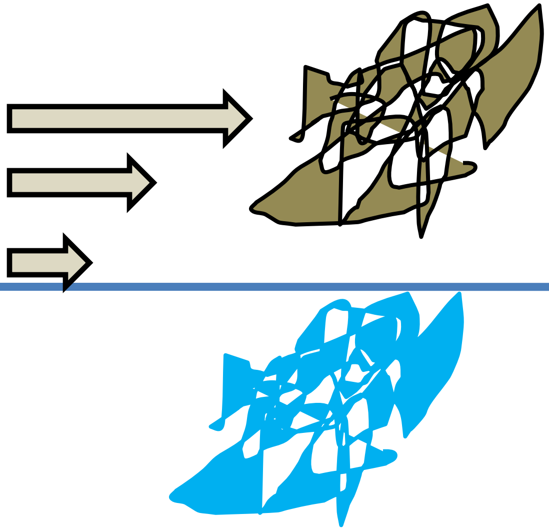
$$\text{Reynolds Number} = \frac{\text{II}}{\text{IV}}$$

Surface Wind Stress τ

(Unstratified Boundary Layer Flow)

Definition: Stress = force per unit area on a parallel surface

wind



$$\frac{\vec{\tau}_{air}}{\rho_{\mathrm{air}}} = -<u_{air}'w_{air}'>$$

Air

Water

$$\frac{\vec{\tau}_{w}}{\rho} = -<u'w'>$$

What is the relationship between $\vec{\tau}_w$ and $\vec{\tau}_{air}$?

$$\boxed{\vec{\tau}_w = \vec{\tau}_{air}}$$

Empirical Formula for Surface Wind Stress Drag C_D Coefficient

$\tau = \rho C_D U_{10}{}^2$ where U_{10} is the wind speed 10m above the water

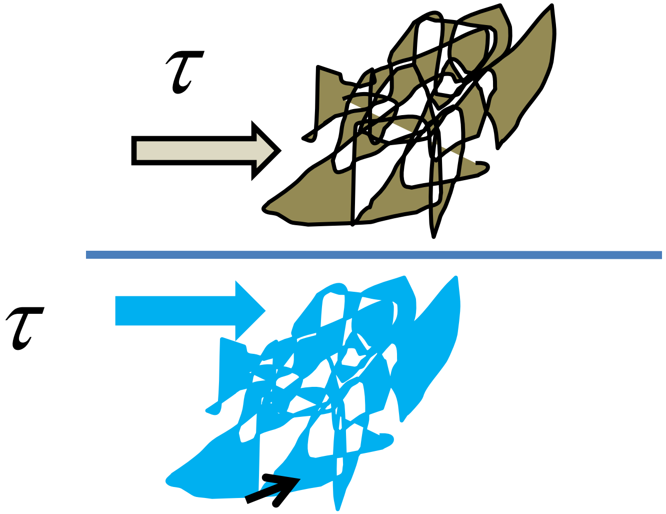
$$C_D = \begin{cases} 10^{-3} & \mathrm{U}_{10} < 5\dfrac{\mathrm{m}}{\mathrm{sec}} \\ 2.5 \cdot 10^{-3} & \mathrm{U}_{10} > 5\dfrac{\mathrm{m}}{\mathrm{sec}} \end{cases}$$

Concept of Friction Velocity u*

Definition

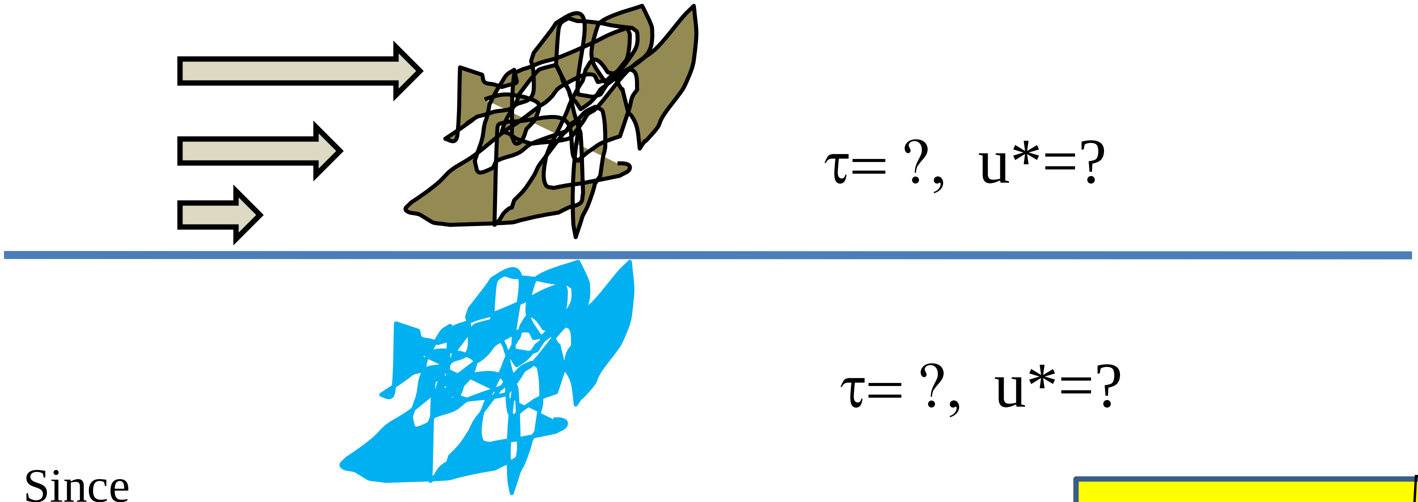
$$\frac{\tau}{\rho} = -<u'w'>= (u^*)^2$$

$$u^* = \sqrt{\frac{\tau}{\rho}}$$



u* Characteristic velocity of the turbulent eddies

Example. If the wind at height of 10m over the ocean surface is 10 m/sec, calculate the stress at the surface on the air side and on the water side. Estimate the turbulent velocity on the air side and the water side.



τ= ?, u*=?

τ= ?, u*=?

Since

$$U_{10} > 5\frac{\text{m}}{\text{sec}} \Rightarrow \quad C_D = 2.5\cdot 10^{-3}$$

$$\tau = \rho_{air} C_D (U_{10})^2 = (1.0\frac{kg}{m^3})(2.5\cdot 10^{-3})(10\frac{\text{m}}{\text{sec}})^2$$

$$\tau = \; .25\frac{\text{N}}{\text{m}^2} = \tau_{air} = \tau_{water}$$

$$\text{u}^*_{\text{air}} = \sqrt{\frac{\tau_{air}}{\rho_{air}}} = \sqrt{\frac{.25\frac{\text{N}}{\text{m}^2}}{1.0\frac{kg}{m^3}}} = .5\frac{m}{\text{sec}}$$

$$\text{u}^*_{\text{w}} = \sqrt{\frac{\tau_{w}}{\rho_{w}}} = \sqrt{\frac{.25\frac{\text{N}}{\text{m}^2}}{1000\frac{kg}{m^3}}} = .016\frac{m}{\text{sec}}$$

Vertical Turbulent Friction

(2D Horizontal Flow Unstratified Boundary Layer Flows)

Horizontal Equations (x, y)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \mathrm{v}\frac{\partial u}{\partial y} - f\mathrm{v} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right) + \frac{1}{\rho}\frac{\partial \tau_x}{\partial z}$$

$$\frac{\partial \mathrm{v}}{dt} + u\frac{\partial \mathrm{v}}{\partial x} + \mathrm{v}\frac{\partial \mathrm{v}}{\partial y} + fu = -\frac{1}{\rho}\left(\frac{\partial p}{\partial y}\right) + \frac{1}{\rho}\frac{\partial \tau_y}{\partial z}$$

Short hand form

$$\frac{D\vec{u}}{Dt} + \vec{f} \times \vec{u} = -\frac{1}{\rho}\nabla_h p + \frac{1}{\rho}\frac{\partial}{\partial z}\vec{\tau}$$

where $\frac{\vec{\tau}}{\rho} = -<\vec{u}'w'>$ & $\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{u}\cdot\nabla$

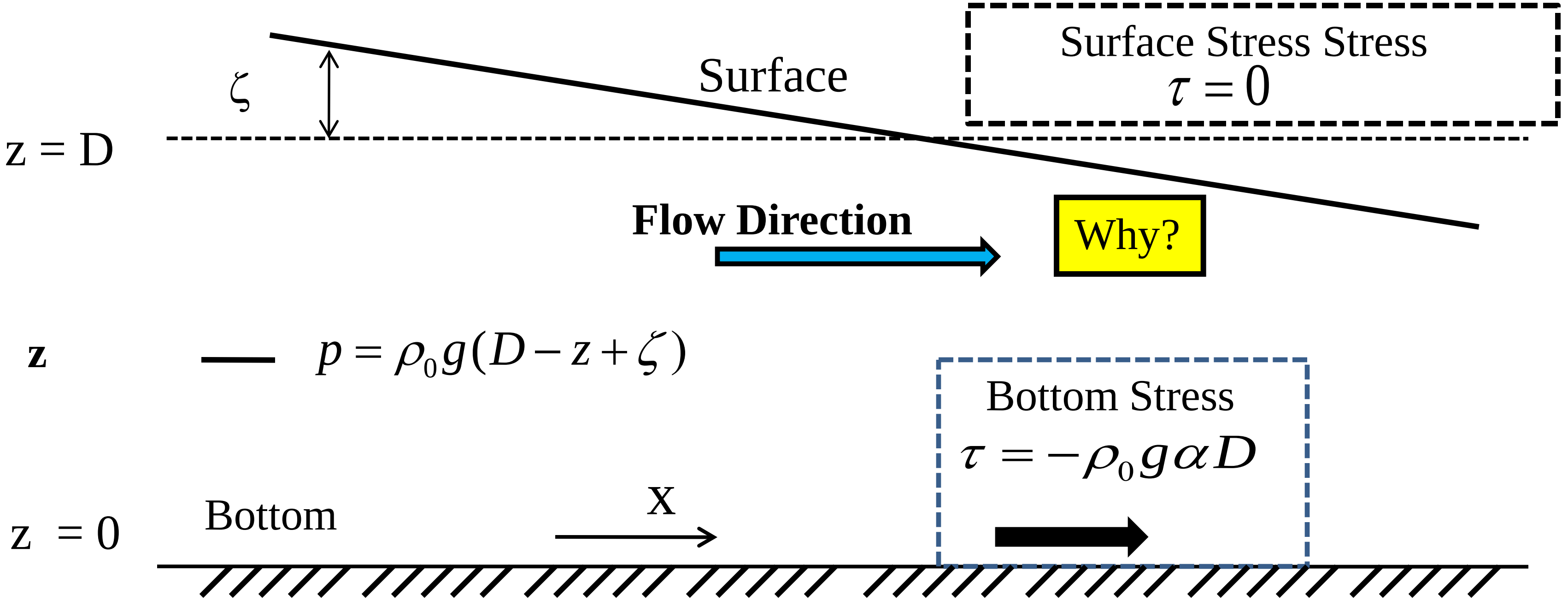
Vertical Equation Hydrostatic condition

$$\frac{\partial p}{\partial z} = -\rho g$$

Role of Bottom Stress

Example : Steady State, Narrow Channel flow (f =0), constant surface slope , α, no wind.

Example : Steady State Channel flow with a constant surface slope , α (No wind)



Surface Stress Stress

$\tau = 0$

Surface

ζ

z = D

Flow Direction

Why?

z $p = \rho_0 g(D - z + \zeta)$

Bottom Stress

$\tau = -\rho_0 g \alpha D$

x

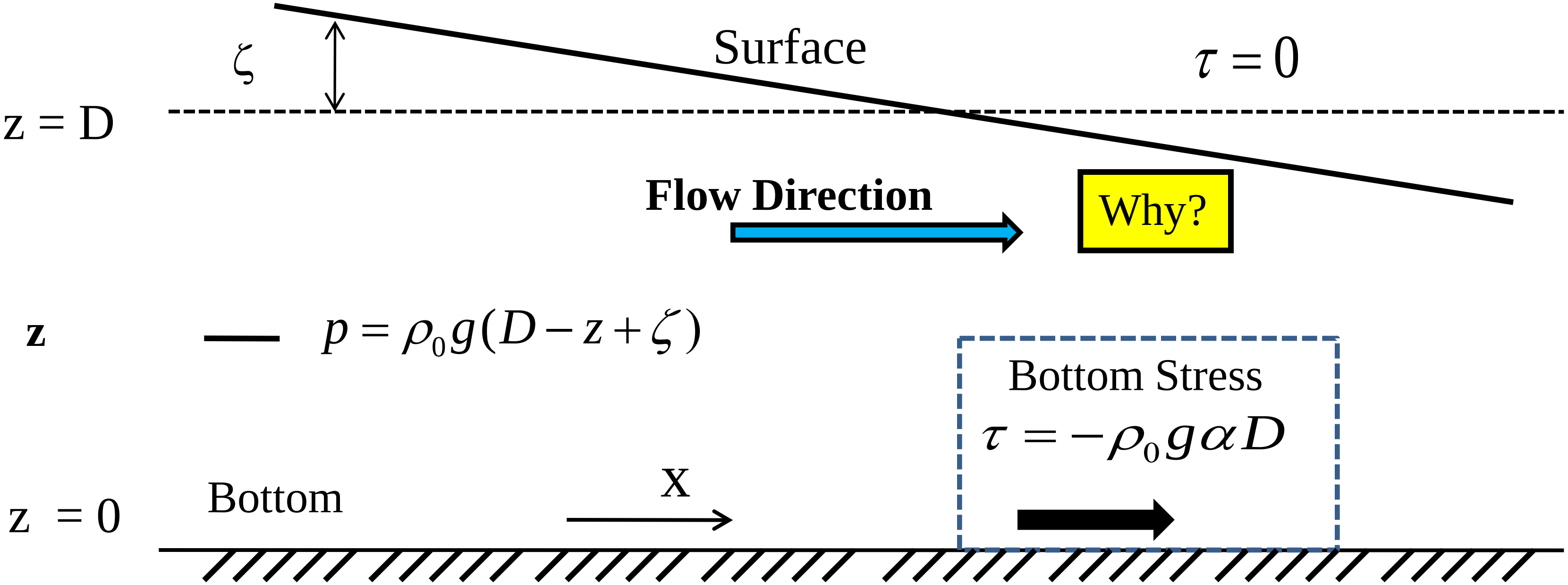
z = 0

Bottom

$$0 = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{1}{\rho_0}\frac{\partial}{\partial z}\tau \qquad \text{but } \frac{\partial p}{\partial x} = \frac{\partial\{\rho_0 g(D - z + \zeta)\}}{\partial x} = \rho_0 g \alpha$$

$$\Rightarrow \tau = -\rho_0 g \alpha (D - z)$$

Note $\alpha = \dfrac{\partial \zeta}{\partial x} < 0$



ζ

Surface

$\tau = 0$

z = D

Flow Direction

Why?

z

$p = \rho_0 g(D - z + \zeta)$

Bottom Stress

$\tau = -\rho_0 g \alpha D$

x

Bottom

z = 0

Typical Values

$$\tau = -\rho_0 g \alpha (D - z) \qquad \alpha = \frac{\partial \zeta}{\partial x} < 0$$

$|\alpha| \approx 1cm/(1km \ to \ 10km) = 10^{-5}$ to 10^{-6} & for D =10m

$\Rightarrow$ Friction velocity on the bottom is

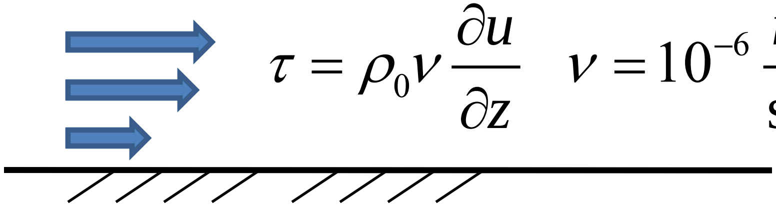
$$u^* = \sqrt{\frac{\tau}{\rho_0}} = \sqrt{g\,|\alpha|\,D}$$

$$\Rightarrow \ u^* = (1-3)cm/\sec$$

Relating Stress to Velocity

Viscous (molecular) stress in boundary layer flow
Low Reynolds Number Flow

Note: Viscous Stress is proportional to shear.



$$\tau = \rho_0 \nu \frac{\partial u}{\partial z} \qquad \nu = 10^{-6}\,\frac{m^2}{\sec}$$ molecular viscoisty

Turbulence Case: Eddy Viscosity Assumption

$$\tau = \rho_0 \kappa \frac{\partial u}{\partial z}$$ κ eddy viscosity

Note. At a fixed boundary $u = 0$ because of molecular friction.

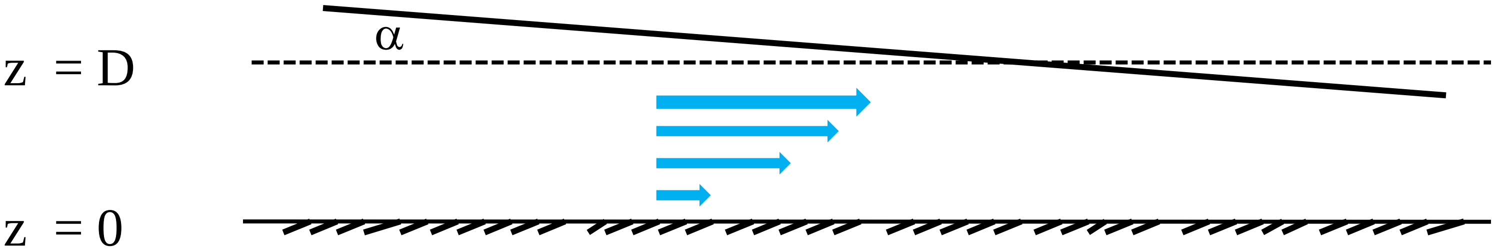
In general κ = κ(z).

Mixing Length Theory: Modeling κ

$$\kappa = \tilde{u} l$$

l a characteristic length , $\tilde{u}$ a characteristic velocity of the turbulence

Back to constant surface slope example where we found that $\tau = -\rho_0 g\alpha(D-z)$



If we use the eddy viscosity assumption with constant k

$$\tau = \rho_0 k\frac{\partial u}{\partial z} = -\rho_0 g\alpha(D-z) \Rightarrow$$

$$u = -\frac{g\alpha z}{k}(D-\frac{z}{2})$$

$$= \frac{(u^*)^2}{k}z(1-\frac{z}{2D})$$

u* = bottom friction velocity

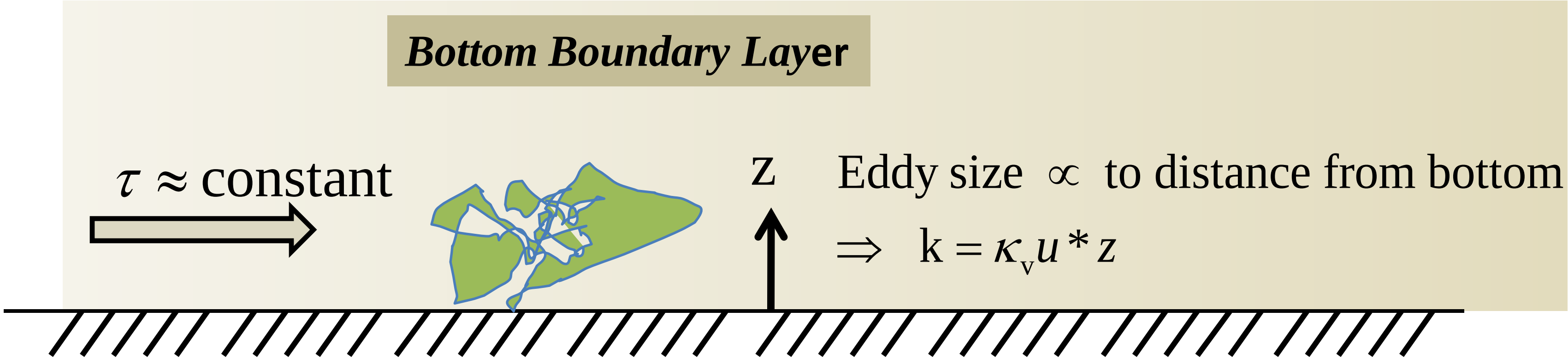
Note we have used the fact that

$$\frac{\partial(z)}{\partial z} = 1 \;\; \& \;\; \frac{\partial(z^2)}{\partial z} = 2z$$

$$\& \; (u^*)^2 = \frac{\tau}{\rho_0} = -g\alpha D$$

Log Layer

Note: in the previous example near the bottom, τ independent of z



Bottom Boundary Layer

$\tau \approx$ constant

z

Eddy size $\propto$ to distance from bottom

$\Rightarrow \quad \mathrm{k} = \kappa_{\mathrm{v}} u^* z$

$$\tau = \rho_0 (u^*)^2 = \rho_0 \kappa \frac{\partial u}{\partial z}$$

$$\Rightarrow \frac{\partial u}{\partial z} = \frac{u^*}{\kappa_{\mathrm{v}} z}$$

Note we have used the fact that

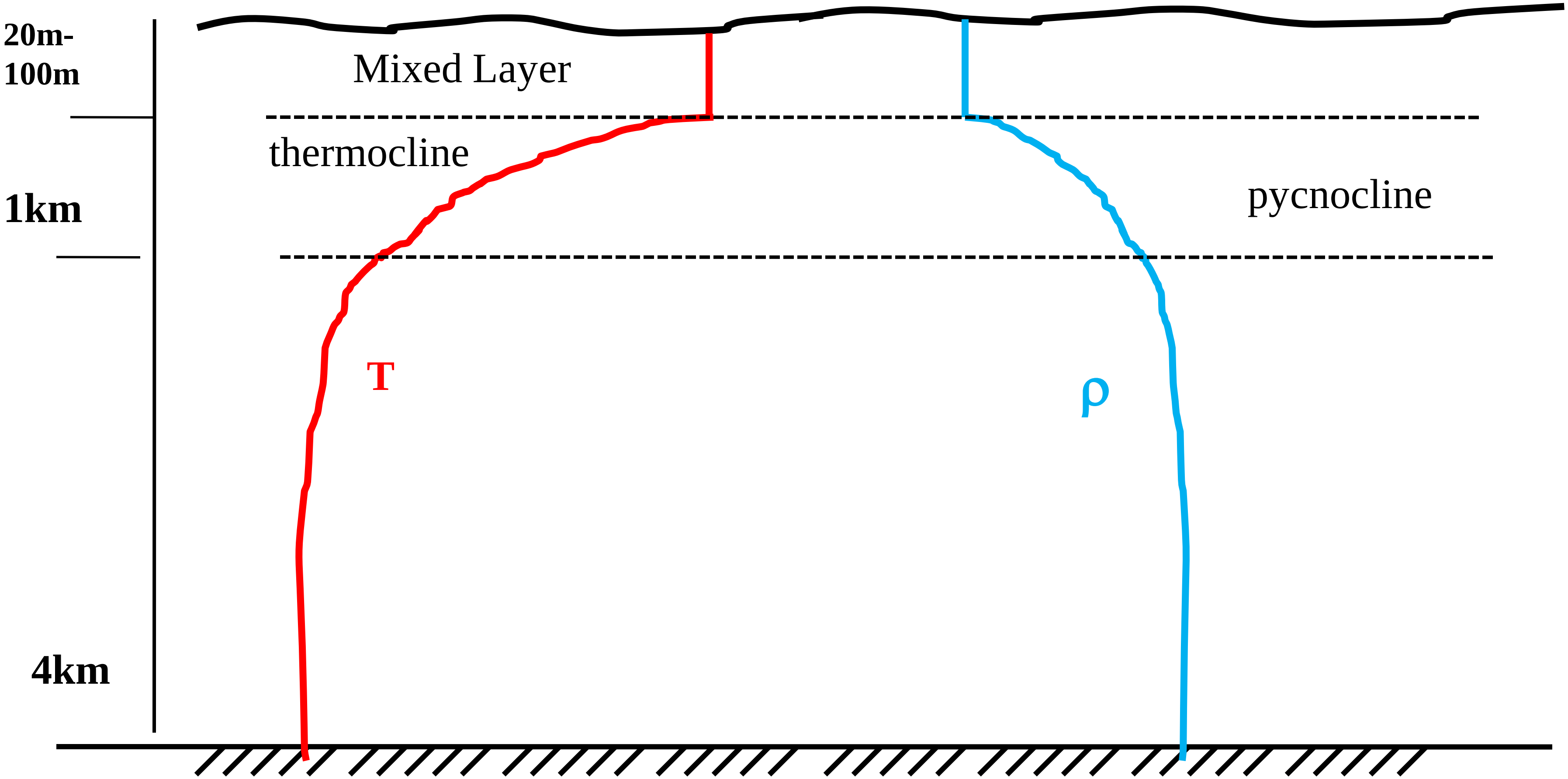
$$\frac{\partial \ln(\frac{z}{z_0})}{\partial z} = \frac{1}{z}$$

$\kappa_{\mathrm{v}} = .4,$ Von Karman's constant

$$\boxed{u = \frac{u^*}{\kappa_{\mathrm{v}}} \ln(\frac{z}{z_0})}$$

z_0 the roughness parameter

Typical Ocean Profile of temperature (T), density (ρ)



20m-
100m

Mixed Layer

thermocline

1km

pycnocline

T

ρ

4km

But $\rho = \rho(T, S, p)$

Stratified Flow

Horizontal Equation

$$\frac{D\vec{u}}{Dt} + \vec{f} \times \vec{u} = -\frac{1}{\rho}\nabla_h p + \frac{1}{\rho}\frac{\partial}{\partial z}\vec{\tau}$$

where $\frac{\vec{\tau}}{\rho} = -<\vec{u}'w'>$ & $\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{u}\cdot\nabla$

Vertical Equation:
Hydrostatic condition
No stratification

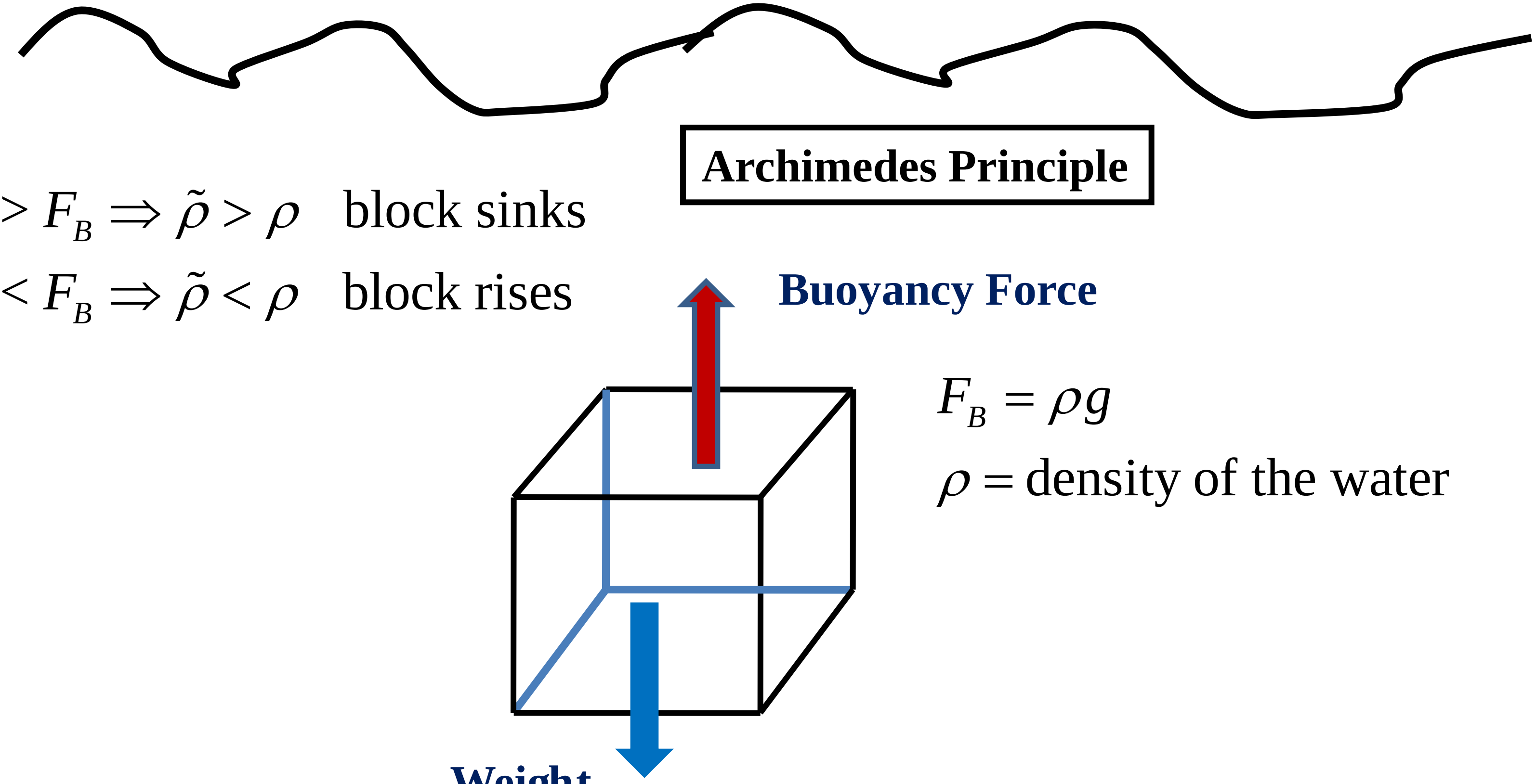
$$\frac{Dw}{Dt} = 0 = -\frac{1}{\rho_0}\frac{\partial}{\partial z}p + g$$

Vertical Equation:
Hydrostatic condition
Stratification

$$\frac{Dw}{Dt} = 0 = -\frac{1}{\rho_0}\frac{\partial}{\partial z}\tilde{p} + \frac{(\rho - \rho_0)}{\rho_0}g$$

$$\tilde{p} = p - \rho_0 g z$$

Buoyancy



Archimedes Principle

If W> $F_B \Rightarrow \tilde{\rho} > \rho$ block sinks

If W< $F_B \Rightarrow \tilde{\rho} < \rho$ block rises

Buoyancy Force

$$F_B = \rho g$$

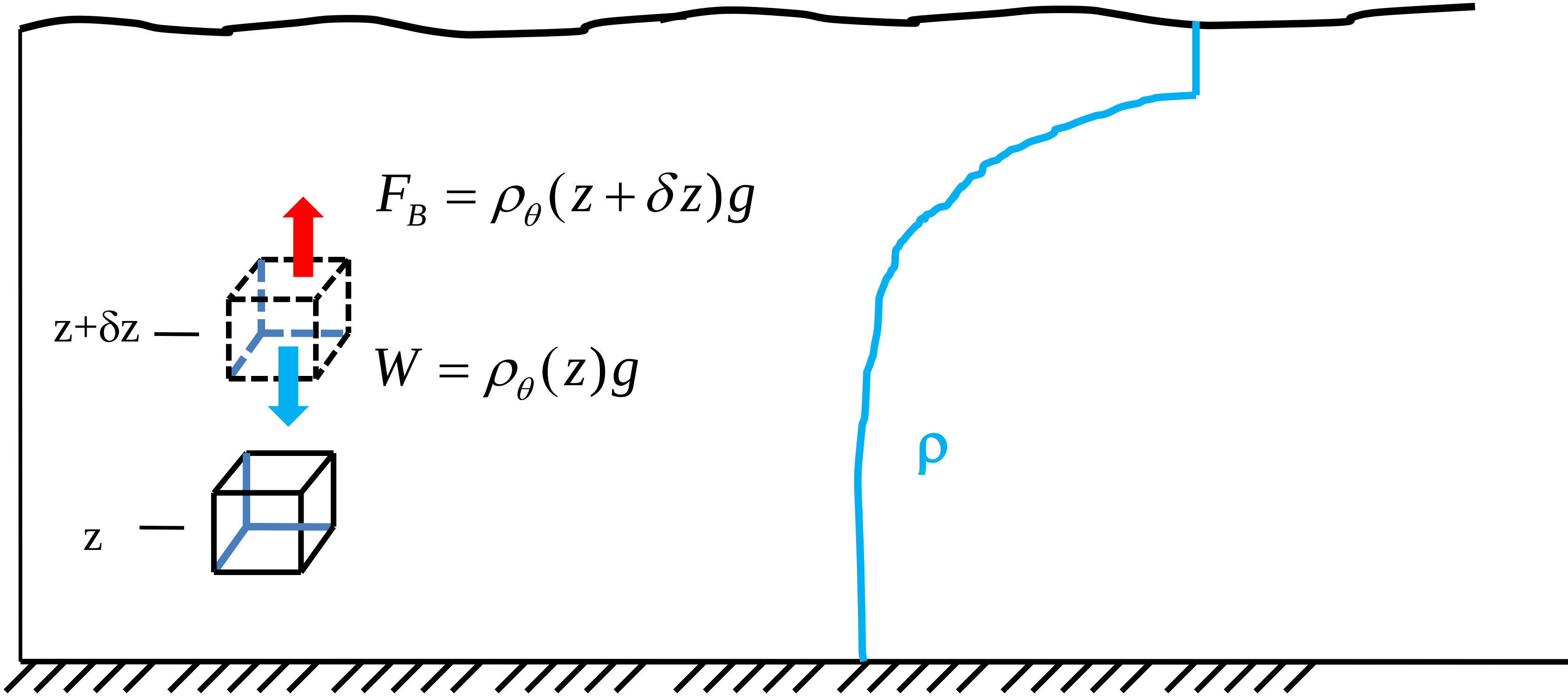
$\rho =$ density of the water

Weight

$$W = \tilde{\rho} g$$

$\tilde{\rho} =$ density of the block

Concept of Buoyancy frequency N



$$F_B = \rho_\theta(z+\delta z)g$$

$$W = \rho_\theta(z)g$$

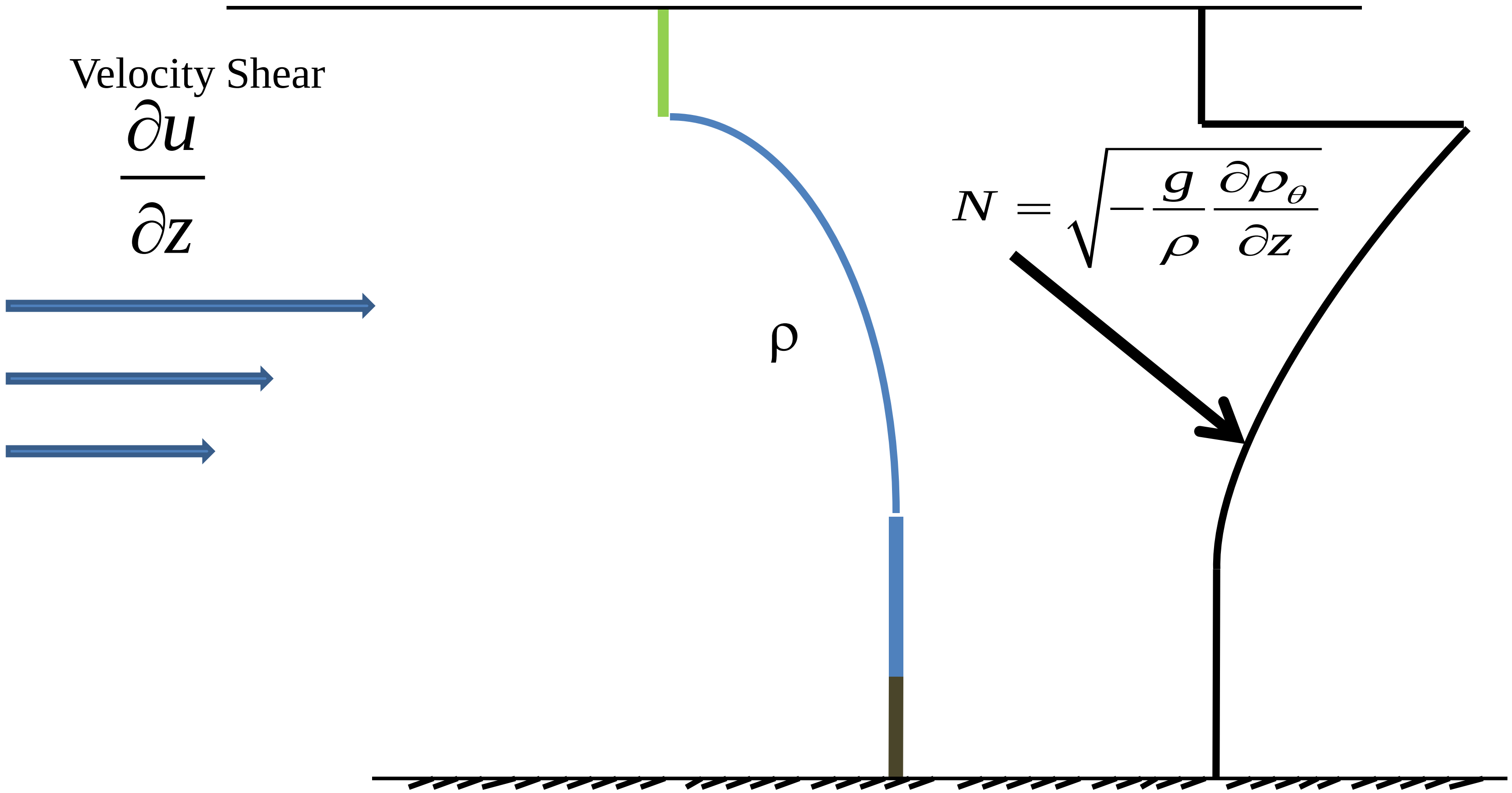
$$F_{net} = F_B - W = V\{\rho_\theta(z+\delta z) - \rho_\theta(z)\}g$$

but $\dfrac{\partial \rho_\theta}{\partial z} \approx \dfrac{\{\rho_\theta(z+\delta z) - \rho_\theta(z)\}}{\delta z}$

$F_{net} = V\dfrac{\partial \rho_\theta}{\partial z}\delta z g = -V\rho N^2 \delta z$ where $N^2 = -\dfrac{g}{\rho}(\dfrac{\partial \rho_\theta}{\partial z}) = \{-\dfrac{g}{\rho}\dfrac{\partial \rho}{\partial z} + \dfrac{g^2}{c^2}\}$

$$\frac{g^2}{c^2} \approx 4.4 \cdot 10^{-5}\ \text{sec}^{-2}$$

Turbulence in the Pycnocline



Velocity Shear

$$\frac{\partial u}{\partial z}$$

$$N = \sqrt{-\frac{g}{\rho}\frac{\partial \rho_\theta}{\partial z}}$$

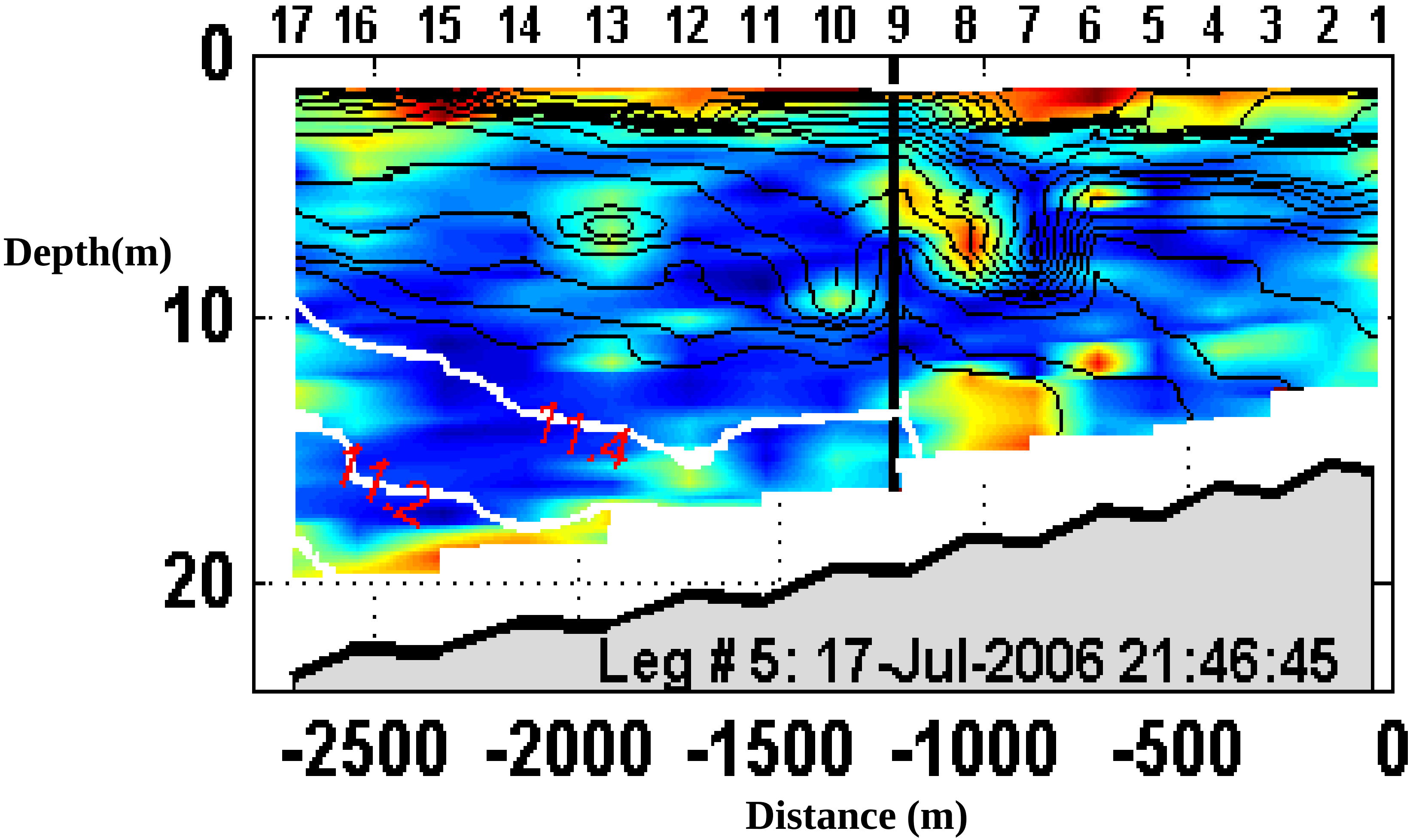
Gradient Richardson Number

$$Ri_g = \frac{N^2}{(\frac{\partial u}{\partial z})^2}$$

Turbulence occurs when

$$Ri_g < \frac{1}{4}$$

Billow clouds showing a Kelvin-Helmholtz $(Ri_g < \frac{1}{4})$ instability at the top of a stable atmospheric boundary layer. Photography copyright Brooks Martner, NOAA Environmental Technology Laboratory.



Turbulence Observed in an internal solitary wave resulting in $(Ri_g < \frac{1}{4})$
Goodman and Wang (JMS, 2008)

Temperature (Heat)Equation with Molecular Diffusion

$$\frac{dT}{dt} = \cancel{\kappa_T \nabla^2 T}$$

where

$$\frac{d}{dt} = u\frac{\partial}{\partial x} + \mathrm{v}\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

$$\kappa_T = \text{molecular diffusion}$$

$$= 1.4 \cdot 10^{-7}\,\frac{m^2}{\text{sec}}$$

Approach for Turbulence

$$\frac{dT}{dt} = 0$$

But $T = \overline{T} + T'$

$$\Rightarrow$$

$$< \frac{\partial T}{\partial t} > + < u\frac{\partial T}{\partial x} > + < \mathrm{v}\frac{\partial T}{\partial y} > + < w\frac{\partial T}{\partial z} >= 0$$

$$\frac{\partial \overline{T}}{\partial t} + \cancel{\overline{u}\frac{\partial \overline{T}}{\partial x}} + \cancel{\overline{\mathrm{v}}\frac{\partial \overline{T}}{\partial y}} + \overline{w}\frac{\partial \overline{T}}{\partial z} + \frac{\partial}{\partial z} < w'T' >= 0$$

Case of Vertical Advection and Turbulent Flux

$$\frac{\partial \overline{T}}{\partial t} + \overline{w}\frac{\partial \overline{T}}{\partial z} + \frac{\partial}{\partial z} < w'T' >= 0$$

Eddy Diffusivity Model

$$< w'T' >= -k_T\,\frac{\partial \overline{T}}{\partial z}$$

Advection Diffusion Equation

drop bar notation

$$\frac{\partial T}{\partial t} + w\frac{\partial T}{\partial z} - \kappa_T \frac{\partial^2 T}{\partial z^2} = 0$$

Steady State Case $\frac{\partial T}{\partial t} = 0$

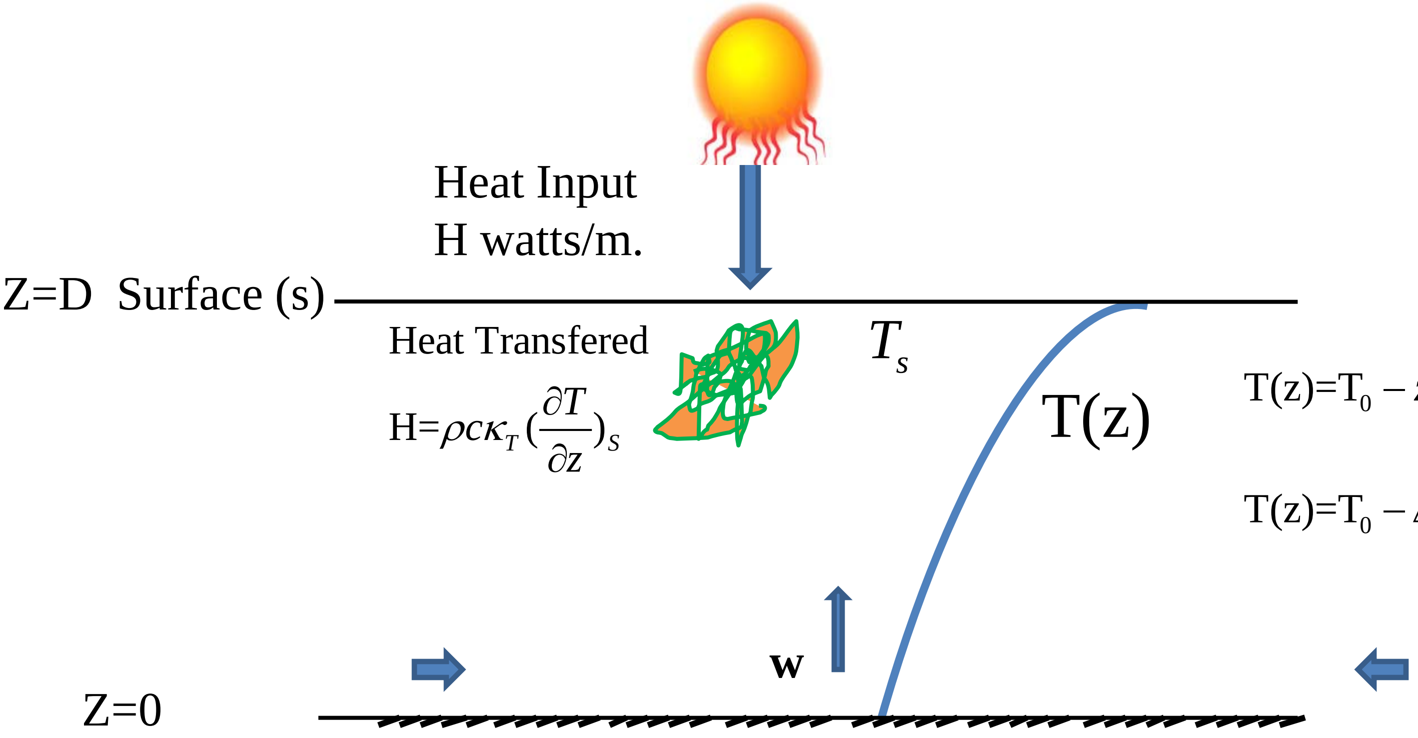
$$w\frac{\partial T}{\partial z} - \kappa_T \frac{\partial^2 T}{\partial z^2} = 0 \quad \overline{w} = upwelling\ \ velocity$$

$Solution:$

$$\beta = \frac{\partial T}{\partial z} = A\exp(\frac{w}{\kappa_T}\tilde{z}) \quad \tilde{z} = D - z$$

$$= (\frac{\partial T}{\partial z})_0 \exp(\frac{\tilde{z}}{z_0})$$

$$z_0 = \frac{\kappa_T}{w}$$



Heat Input
H watts/m.

Z=D Surface (s)

Heat Transfered

$H=\rho c\kappa_T(\frac{\partial T}{\partial z})_S$

T_s

T(z)

$$T(z)=T_0 - z_0(\frac{\partial T}{\partial z})_s\{1-\exp(-\frac{(D-z)}{z_0})\}$$

$$T(z)=T_0-\Delta T\{1-\exp(-\frac{D-z}{z_0})\}$$

w

Z=0

Note: $(\frac{\partial T}{\partial z})_0 > 0$

$$c = \text{specific heat of water } 4.2\cdot 10^3 \frac{J}{oC-kg}$$