Acoustic scattering from a thermally driven buoyant plume revisited

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ABSTRACT

Far-field weak scattering theory is applied to the case of high frequency broadbandwidth acoustic scattering from a thermally generated buoyant plume in a controlled laboratory environment. To first order, the dominant scattering mechanism is thermally driven sound speed variations that are related to temperature deviations from ambient. As a result, the received complex acoustic scattering is a measure of the one-component three-dimensional Fourier transform of the temperature difference field measured at the Bragg wave number. The Bragg wave number vector is the difference between the scattered and incident wave vectors. Solving for its magnitude yields the Bragg scattering condition; this is the Fourier component of the plume variability that produces scattering. Results are presented for multi-static scattering from unstable and turbulent plumes using a parallel scattering geometry. The data justify application of the far-field weak scattering theory to the present case of a thermal plume. As a consequence, quantitative results on medium variability can be inferred using high frequency broadbandwidth acoustic scattering. Particular attention is given to the role of anisotropy of the variability of the scattering field in determining the validity of far field Bragg scattering.

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I. INTRODUCTION

High frequency broadbandwidth acoustic scattering in water is of considerable interest to the physical and biological oceanographic communities. Since sound traveling through a medium will scatter due to impedance changes, it is of interest to explore whether it is possible to exploit these changes as a means to remotely measure medium variability such as fluctuations in the density, sound speed, or fluid velocity. Where these variations are such that weak scattering theory can be applied to describe the behavior of the sound, and that the scattering volume satisfies the far-field condition, quantitative spatial information of the medium variability can be obtained from the acoustic scattering process.[1, 2] Factors such as the presence of particulates and biologics in the water column also contribute to scattering and impede interpreting the received acoustic scattering signal just in terms of medium variability. Scattering from particulates can have similar and many times greater magnitude scattering strengths than that from impedance fluctuations of the medium. Thus relying on signal level alone to determine the source of scattering can lead to interpreting incorrectly the acoustic data in terms of the type of scattering occurring. To avoid the uncertainty in knowing the source of the scattering, detailed knowledge of the scattering volume and scattering process is necessary. This information is also necessary to ground-truth the acoustic scattering; however, such ground truth information is often difficult to acquire.

Possibly the first calibrated *in situ* acoustic scattering experiment with accompanying environmental data was reported recently by Seim *et al*.[3] The experiment used downward looking 120 and 200 kHz calibrated transducers in a salt-stratified tidal channel near Puget Sound, Washington. The environment contained energetic turbulent mixing events and was a near ideal location to investigate acoustic scattering from turbulence and to make comparisons with environmental measurements. An advanced microstructure profiler was used to measure χ_T , the turbulent thermal variance dissipation rate and ε , the dissipation rate of turbulent kinetic energy. A Batchelor spectrum was employed to extend the measured temperature gradient spectrum beyond sensor capabilities. This allowed a spectral comparison between the environmental measurements and the acoustic scattering measurements at the same wave numbers. Seim et al.[3] found instances when the environmentally measured turbulence was large enough to account for the observed acoustic scatter. For the most part, however, the observed scattering strengths were higher than what the measured turbulent levels predicted. In most instances the scattering was likely due to particulates and biologics in the water column. Although great effort was taken to align the environmental and acoustic data sets (both spatially and temporally), unequal sampling volume sizes between the acoustically derived and oceanographically derived estimates of scattering strength led to the greatest source of uncertainty in these comparisons.

High frequency acoustic scatter from biologics is capable of making biological biomass abundance estimates provided supporting information such as classification of scatterer, number density, and size distributions can be obtained to calibrate the acoustic data. More complex use of the acoustic scatter to infer scatterer dimensions and bulk material parameters is possible through application of various scattering models.[4-10] When applied in the forward direction, material parameters determined from independent measurements are used as inputs for the models and the scattering function thus obtained can be compared to that derived from the acoustic scattering. The amount of agreement depends upon the accuracy of the model, the accuracy of the material parameters used, and the ability to sample, both environmentally and acoustically, the same volumes. When the scatterer is known, and the scattering mechanism is well understood and accurately modeled, the acoustic data can be inverted to obtain quantitative information on the scatterers.[11-12] Similarly, high frequency acoustic scatter from suspended sediments in the water column can be used to track regions of particulates, and in some instances, can yield quantitative information such as particle size and number density.[13-17]

Acoustic scatter from small scale biologics and suspended sediments can be on the same order of magnitude as regions of intense turbulence.[3,4,18,19] A first step towards classification of the source of the acoustic scatter is to understand the scattering properties for each of the possible sources. This manuscript focuses on examining the acoustic scatter from thermally generated turbulence in a controlled laboratory environment. The equation governing the acoustic scatter has been well established from first principles for a wave traveling through a region of medium variabilty.[20,21] The manuscript reports on the application of far-field weak scattering theory to the case of high frequency broadband acoustic multi-static scattering from a thermally generated buoyant plume.

Two previous investigations reported on the frequency and angular dependence of scattering for a cylindrically symmetric laminar plume[1] case and on the use of a multi-static scattering geometry for the unstable and turbulent plume cases.[2] The first set of measurements showed that the acoustic scattering to be the result of thermally generated variability in the index of refraction. The laminar plume experiment resulted in a confirmation of the far-field weak scattering theory with the received complex acoustic scatter proportional to the one-component two-dimensional Fourier transform of the scattering field. An important consequence of far-field weak scattering theory is the Bragg scattering condition, which is described in detail in Sec. II. This condition expands the upper and lower limits of the spatial frequencies of the scattering field being observed by utilizing simultaneous multi-static measurements with common Bragg wave numbers. It should be noted that this is not to be confused with the spatial frequency resolution or Bragg wave number resolution, which is a function of the transmitted signal.

The Bragg scattering condition also provides an indirect means of verifying the approximations made to the scattering theory. For the specific case of the laminar plume, the far-field and weak scattering approximations used in the theory resulted in good agreement between the measured acoustic scatter and a model prediction that used independent temperature measurements.

For the more complex case of the turbulent plume, the environmental measurements required for intercomparison are difficult to obtain. This is because of the need to acquire simultaneous acoustic and environmental data over the same spatial region and at the resolution of order the wavelength of the acoustics. An intercomparison can be made, however, by using the Bragg scattering condition. The Bragg scattering condition leads to a technique whereby the received complex acoustic scatter obtained from different scattering angles can be compared in regions of equivalent/overlapping Bragg wave numbers. The far-field weak scattering theory predicts that the magnitude of the acoustically estimated spectra obtained at different scattering angles are equal at equal Bragg wave numbers.

Unlike the case for the laminar plume, early results from the turbulent plume measurements did not confirm this prediction, thus suggesting a problem in the application of far-field weak scattering theory to the turbulent plume. An anti-parallel scattering geometry experiment was designed to investigate specifically the cause of the failed prediction.[2] The results showed that wave front curvature effects must be taken into account to describe properly the instantaneous scattering from the turbulent plume. Two undesirable consequences of this are: (1) that the received acoustic scatter no longer represents a one-component Fourier transform of the scattering field; and (2) that the Bragg scattering condition cannot be used as a substantiation of the scattering theory. Because the scattering from the turbulent plume is more similar to near-field than far-field scattering, the quantitative information of medium variability contained in the acoustic scatter appeared to be limited.

However, if instead of examining the instantaneous received pressure field associated with scattering from the turbulent plume, the ensemble average of the intensity is used, the Bragg scattering condition may be recovered. This results when the variability field is approximately statistically isotropic on the scale of the inverse of the Bragg wave number. The scattered field can then be interpreted in terms of the wave number spectrum of the temperature field. It should be noted that for this case, since the scattering field is distributed throughout the scattering volume and the contributions at the Bragg wave number are approximately statistically isotropic, the effective "average" spatial scale of scattering is much smaller than the Fresnel radius.

In Section II the equation describing the acoustic scatter is given for the case of far-field weak scattering theory. This is followed by a description of the Bragg scattering condition. Subsections include discussions of near-field and beam pattern effects on the scattering process and the usage of ensemble averaged spectrum for isotropic variability to recover the Bragg scattering condition. Section III describes the experimental setup, laboratory procedures and the measurements that are taken. In Sec. IV the results are given for two dynamically different thermal plumes. These are described as the unstable and turbulent thermal plumes. The summary and conclusions are given in Sec. V.

II. THEORETICAL BACKGROUND

High frequency broadbandwidth acoustics are well suited for studying volume variability in the ocean on mm to cm scales. The governing equation of motion is developed by considering the behavior of a sound wave as it travels through a homogeneous region and encounters a localized anomaly.[1,2,20-23] This is expressed as the inhomogeneous Helmholtz equation with a source term representing the scattering due to the anomalous region. The source term is expressed as a function of three fundamental quantities: the relative compressibility, the density, and the fluid velocity. The case considered in this manuscript is that of acoustic scattering from a thermal plume where contributions to the acoustic scatter from density and fluid velocity variability can be neglected, this is shown in Appendix A. The compressibility or index of refraction changes are due to temperature variations of the thermally generated plume, to within an order of magnitude.

The complex acoustic scatter can be described analytically through the integral form of the Helmholtz equation. The integral contains the product of the pressure field, the relative compressibility, and the Green's function; the volume of the integral is defined by the source-receiver beam patterns. When the product of the relative compressibility, the acoustic wave number, and the length scale typical of the anomalous region within the scattering volume is small[22], weak scattering theory is satisfied and the incident field, which can be measured at the volume, replaces the unknown total pressure field in the scattering volume. Simplification of the integral expression occurs when the scattering volume is in the far field of the source transducer. This means that in this circumstance the acoustic waves incident upon the scattering volume are approximated as planar.

As a consequence of using far-field weak scattering theory, the received complex acoustic scatter can be written in terms of a parameter "*m*" that denotes the spatial symmetry or dimensionality of the scattering process associated with the geometrical spreading of the scattered waves. This results by noticing the similarities that exist between the one, two, and three-dimensional Green's function solutions to the Helmholtz equation in the far field. The expression for the far-field approximation of the Green's function solution to the Helmholtz equation in *m*-dimensions, where $m = 1$, 2 or 3 can be written

$$g_m(\mathrm{r},\boldsymbol{r}') = (\pi r)^{(1-m)/2}\, 2^{-(1+m)/2}\, k^{(m-3)/2}\, exp[ikr - i\boldsymbol{k}_s \cdot \boldsymbol{r}' + i\pi(3-m)/4]\ . \tag{1}$$

The coordinate origin is defined at the center of the scattering volume; r is the distance from the origin to the receiver; $\boldsymbol{r}'$ is the vector from the origin to the scattering point; k is the acoustic wave number; and $\boldsymbol{k}_s$ is the scattered wave vector. Using the same theoretical development for the acoustic scatter as presented in a previous paper with the exception of replacing the 3-dimensional Green's function [J. Acoust. Soc. Am., **100**, 1451-1462, (1996), Eq. (6)] with the *m*-dimensional form, the complex acoustic scatter in the frequency domain, neglecting the phase is

$$p_s(\omega) = p_o(\omega)(2\pi r)^{(\frac{1-m}{2})} k^{(\frac{1+m}{2})} \int B(\mathbf{x}')\mu(\mathbf{x}')exp[-i\mathbf{K}\cdot\mathbf{x}']d^m\mathbf{x}'. \qquad (2)$$

It is straightforward to verify the resulting equations take on the correct forms when $m=1, 2, or\ 3$. $p_s(\omega)$ and $p_o(\omega)$ are the complex acoustic scatter measured at the receiver and the incident pressure field at the scattering volume; $\omega = ck$ is the angular acoustic frequency. $\mu = \Delta c / c$ is the relative index of refraction; B is the source-receiver beam pattern. K is defined as the Bragg wave number and is the magnitude of the difference between the scattered wave vector and the incident wave vector,

$$K = |\mathbf{k}_s - \mathbf{k}_i|. \qquad (3)$$

The magnitude of the Bragg wave vector is the product of twice the acoustic wave number and the sine of the scattering half angle measured from the forward direction,

$$K = 2k\, sin(\theta / 2)\,. \tag{4}$$

Because of the importance of Eq. (4) in this paper, it is referred to as the Bragg scattering condition.

For a given source-receiver scattering geometry, the Bragg wave vector is aligned in a fixed direction. The notation is simplified by choosing a coordinate system such that the "*x*-axis" is parallel with the Bragg wave vector. This geometry is illustrated in Fig. 1. Note the *x*-axis is the bisector of the interior angle defined by the source to scattering volume and scattering volume to receiver vectors.

The Bragg wave vector number is the Fourier component of the scattering-producing field as indicated in Eq. (2). The focus of this report will deal with the integral in Eq. (2) and its interpretation. Consider

$$\phi_m(K) = \frac{1}{(2\pi)^m} \int B(\mathbf{x}')\mu(\mathbf{x}')\, exp[-iKx']\, d^m\mathbf{x}'\,. \tag{5}$$

where we will use either $m = 2$ or 3 for the two and three-dimensional scattering cases. The integral in Eq. (5) is written for the special case when the *x*-axis is aligned with the Bragg wave number direction. Thus for this case $\phi_m(\mathbf{k})=\phi_m(k_x,k_y,k_z)$ with $k_x=K$, $k_y=k_z=0$. The beam pattern function *B* is taken as a smooth weighting to the integrand in

Eq. (5), which is actually an approximation to the one-component, *m*-dimensional Fourier transform of the index of refraction fluctuations. The wave number resolution is proportional to the inverse of the effective scattering volume defined by B. The acoustical estimate of ϕ_m, denoted by γ_m, is determined by rearranging terms in Eqs. (2), (4), and (5) yielding

$$\begin{aligned}\gamma_m(K) &= \frac{1}{(2\pi)^{\mathrm{m}}} h(\omega)(2\pi r)^{(\frac{m-1}{2})}(2\sin\frac{\theta}{2})^{(\frac{m+1}{2})} K^{-(\frac{m+1}{2})} \\ &= \phi_m \end{aligned}, \tag{6}$$

where $h(\omega) \equiv p_s(\omega) / p_o(\omega)$ is the transfer function of the scattering process. The relation between the Fourier transform of the temperature field and that of the index of refraction, the case considered in this manuscript, is $\phi = \alpha\phi_{\mathrm{T}}$ where $\alpha = (1/c)(\Delta c / \Delta T)$.

Due to the finite bandwidth of available transducers, $p_i(\omega)$ can provide information over a limited range of acoustic frequencies. For a given acoustic bandwidth, the Bragg wave numbers determined from Eq. (4) and $k = \omega / c$ are similarly limited by the bandwidth of the incident acoustic pulse. This constraint limits the wave number information of Eq. (6). For example, the laboratory measurements reported in this paper use impulsive incident signals with 10 dB down points at 425 and 725 kHz. At backscattering, the available Bragg wave numbers for the stated frequency bandwidth range is between 3607 and 6156 rad/m. The significance of the Bragg scattering condition in Eq. (4) is seen by considering multiple bistatic measurements made in a common direction at a series of scattering angles. A schematic of this scattering geometry is given in Fig. 2 and is called the common Bragg direction configuration. In this arrangement the sources and receivers are placed on the perimeter of a circle and are directed into the axis of symmetry. Each source has a corresponding receiver. Note the parallel Bragg wave vector directions for each source/receiver pair.

One consequence of the broadband Bragg scattering condition is that multi-static scattering is capable of expanding the bandwidth of the acoustic estimate of the index of refraction (temperature) spectrum. Thus when far-field weak scattering theory is satisfied, high frequency broadband multi-static acoustic scattering from medium variability can be used to resolve more fully the wave number transform of the scattering field.

Providing a technique to ground truth these ideas is very difficult. At present there does not exist a quantitative technique to measure the three-dimensional structure of a temperature (and thus, for the case of a buoyant plume, the index of refraction) field over the domain of the scattering volume and at spatial resolutions of the order of or smaller than the Bragg scattering wavelength. The comparison that is performed in this manuscript is between the one-dimensional frequency spectrum obtained from a temperature probe and one derived from the acoustic scattering experiment. Since this is a limited testing of the theory an alternative approach is to examine the constraints imposed by the consequence of Bragg scattering theory itself.

From the definition of ϕ_m given in Eq. (5) and estimated in Eq. (6) by γ_m, γ_m measured at different scattering angles corresponding to the same Bragg wave number must be identical, .i.e.

If $K = K_i \cap K_j$, then $\phi_i(K) = \phi_j(K)$. (7)

That is $\phi_i(K) = \phi_j(K)$ for all *K* which are common to both K_i and K_j; here the subscripts denote measurements made at the i^{th} and j^{th} scattering angles. Equation (7) forms a basis through which the far-field weak scattering theory can be at least indirectly verified when applied to the case of the buoyant plume.

A. Near-field Effects

An earlier paper showed that predictions made by the far-field weak scattering theory do not always hold true when applied to the laboratory case of scattering from a thermally generated buoyant plume.[2] If $K = K_i \cap K_j$, and $\phi_i(K) \neq \phi_j(K)$ then there must be a breakdown in one of the two assumptions used to describe the acoustic scattering. These two assumptions are: (1) the far-field approximation; and (2) weak scattering theory. The condition for weak scattering theory to be valid is given by $\mu kD << 1$ where μ is a characteristic value of the index of refraction, *k* the acoustic wave number and *D* the characteristic value of the length scale of the scattering.[22] For the turbulent plume considered here maximum parameter values are $\mu = 2 * 10^{-2}$, $k = 3 * 10^3$ rad/m, $D = 1 * 10^{-2}$ m, such that $\mu kD = 0.6$ rad. The condition for weak scattering theory to be valid is satisfied, although marginally. This suggests the far-field approximation is the likely source for the inequality in $\phi_i(K) \neq \phi_j(K)$.

When the far-field approximation is not satisfied, higher order terms need to be included in the exponential of Eq. (2). Consider, for simplicity, the case of three-dimensional scattering. Results for the two-dimensional scattering case are entirely analogous. Equation (2) then becomes

$$p_s(\omega) = p_o(\omega)(2\pi)^2 r^{-1} k^2 \Gamma \ , \qquad (8)$$

where

$$\begin{aligned}\Gamma\ (K) &= \frac{1}{(2\pi)^3}\int B(\mathbf{r}')\mu(\mathbf{r}')\exp[-iKx'+ik\zeta]d\mathbf{r}' \\ &= \frac{1}{(2\pi)^3}\int_V \mu(\mathbf{r}')\exp[-iKx'+ik\zeta]d\mathbf{r}'\end{aligned} \qquad (9)$$

and the phase correction term, ζ, to lowest order is given by

$\zeta = \left(x'^2\cos^2(\theta/2) + y'^2\sin^2(\theta/2) + z'^2\right)/r$. Note that γ_m with $m = 3$ of Eq. (6) is the acoustic estimate of Γ. The notation of subscripting the second integral in Eq. (9) with V indicates that the beam pattern weight B is used over the volume domain of integration. The beam pattern acts like a spatial Hanning window in this integral. Note that $\Gamma\ (K)$ no longer represents a Fourier transform of the index of refraction field, because of the presence of the term ζ and thus that the multi-static scattering configuration in a common Bragg wave number direction can no longer be employed to extend the effective bandwidth of the estimated Fourier transform of the scattering field. The quadratic terms in ζ represent wave front curvature or near-field-like correction terms to the far-field approximation.

B. On the Nature of Bragg Scattering from a Turbulent Field

Previous unpublished results for multi-static scattering in a common scattering direction from a turbulent plume showed a breakdown of the prediction of the Bragg scattering condition given in Eq. (7), i.e., $\phi_i(K) \neq \phi_j(K)$ when $K = K_i \cap K_j$. Typically, the far field of a baffled piston is defined as $r / a >> 1$ and $r / a >> ka$ where r is the range to the field point, and a is the radius of the acoustic source.[24] Approximating the acoustic beam radius at the volume as $r_b = r /(ka)$ and the Fresnel radius as $r_F = \sqrt{r\lambda / 2}$, then the previous inequalities can be restated as $r_b >> \lambda$ and $(r_F / r_b)^2 << 1$. This last inequality implies that when the far-field condition is satisfied, there are many Fresnel zones within the volume defined by the acoustic beam. Table 1 shows values of the various parameters for the experiment described in this paper, which uses a cylindrical baffled piston.

The presence of many Fresnel zones on the incident wavefront may or may not be desired, depending upon the relation between the Fresnel radius and the spatial extent of the scattering mechanism in the direction of the Fresnel radius (i.e. perpendicular to the direction of the vector Bragg wavenumber) . For instance, consider measuring the scattering characteristic of a localized, spherical point-like scatterer. In this case, as the range to the scattering volume increases, the Fresnel radius increases, and when compared to the constant length scale of the scatterer, the wavefront over the scattering region appears more planar than spherical Contrast this type of point-like scatterer to one where the scattering mechanism is distributed throughout the entire scattering volume, such as a turbulent plume with index of refraction variability which may not have spherical symmetry at any instant of time In this case, as the scattering volume is located further into the far field, the wave front incident upon the entire scattering volume appears more spherical than planar and in our case to individual (thermal) turbulent eddies which produce the scattering. This is a result of the beam width at the volume increasing with range faster than the Fresnel radius increases with range. This implies that scattering from a distributed field of scatterers (such as from the thermal plume used in the measurements in this manuscript) may be in the near field. This might invalidate the usage of the far-field approximation, the Bragg scattering condition, and the ability to obtain an estimate of the Fourier transform of the scattering field. Far-field scattering requires that the size of the scatterer in the direction of athe Fresnel radius be smaller than the Fresenl radius. Thus the degree of symmetry of a scatter or, in the case of statistical quantities such as turbulent spectra, the degree of anisotropy of an ensemble of scatters will play an important role in determining whether the Bragg scattering condition can be used to develop a Fourier integral relationship between the received acoustic filed and the scattering field. These issues will be discussed in detail in the next section. Note aspects of this have also been discussed by one of the authors in a previous manuscript.[20]

If l is a characteristic scaleof the scattererin the direction perpendicular to the Bragg wavenumberthen the criteria for far-field and Bragg scattering is $l << r_F$. The correlation length of the temperature anomalies for the turbulent plume considered in this paper in the direction perpendiculer to the plane of scattering (the "z" direction) and thus along a Fresnel radius can be qualititatively estimated. Uzing a nominal vertical advection velocity of 3 cm/s result in an estimate of 3 mm < l < 1.5 cm (half width at half max.) . Thus, for the experiment to be described in this manuscript, l can be of order the Fresnel radius (see table 1) and eddies of the turbulent thermal plume do not satisfy the criteria of having a sufficiently small spatial scale in the direction of a Fresnel radius atany instant of time. This is why the previous (unpublished) data set indicated a breakdown of the Bragg scattering condition. However if the variability of the temperature field eddies within the thermal plume are turbulent like at the scale of the Bragg wavelength, then it is expected that the ensemble-averaged received pressure field may result in the far field Bragg scattering condition to be satisified and a Fourier integral recovered. . This can occur because in a turbulent field although individual turbulent eddies may have a particular orientation, on average, their orientation is more random and averaging is expected to be bring down the degree of asymmetry or anisotropy. Note that in classical turblence theory at scales of the inertial subrange and smaller[26]the three dimensional spectrum of the field is isotropic even though individual eddies are not necessarily (and indeed in general are not) spherically symmetric. See for example numerical studies on turbulence in references 2x,2y,2z Thus for isotropic turbulence the far-field scattering condition and the Bragg scattering condition can then be recovered and the spectrum of the temperature field estimated acoustically. Realistic turbulent like fields may have anisotropy but it is still expected that the degree of anisotropy will be less than the degree of assymtery of an individual instantaneous eddy. In the next section criteria for the degree of maximum anisotropy for Bragg scattering to be valid is derived. The reader again is also referred to reference 20 for a discussion of this issue.

Thus for a turbulent field the scale or wavenumber of the scattering will play a critical role in determining this issue of isotropy. If the Bragg wavenumber is of order and/or greater than a characteristic inertial subrange wavenumber[26] it is expected that isotropy or near isaotrpy should hold. Alternatively as we will show in the next section acoustic scattering provides a mean of assessing whether in fact the turbulent field has this level of isotropy.

f kHz	λ mm	k rad/m	r_F cm	r_b cm	n_F	$(r_F/r_b)^2$	ka	r_{min} cm	r_{max} cm
425	3.5	1780	2.1	4.0	3.6	0.3	34	5.0	10.2
525	2.9	2199	1.9	3.2	2.9	0.3	42	6.2	12.6
625	2.4	2618	1.7	2.7	2.4	0.4	50	7.4	15.0
725	2.1	3037	1.6	2.3	2.1	0.5	58	8.7	17.5

Table 1.

Table of frequency dependent values of terms relevant to this section. Acoustic frequency *f*, acoustic wavelength $\lambda = c/f$, sound speed *c* = 1500 m/s, acoustic wavenumber $k = 2\pi/\lambda$, radius of first Fresnel zone $r_F = \sqrt{r\lambda/2}$, *r* = 25 cm, radius of beam at scattering volume (at -10 dB) $r_b = 5.4r/(ka)$, *a* = radius of transducer = 1.9 cm (0.75 in.), number of Fresnel zones $n_F = 2r_F/(r\lambda)\big|_{r_F=r_b}$, first axial near field null coming in from infinity $r_{min} = a^2/(2\lambda) - \lambda/2$, first axial maximum coming in from infinity $r_{max} = a^2/\lambda - \lambda/4$; and also, in this manuscript, r/a = 13.1.

C. Recovering the Bragg Scattering Condition

Using eq.(9) consider the ensemble average of received acoustic intensity

$$
\begin{aligned}
\Psi &= <\Gamma'\Gamma'^{*}> \\
&= \frac{1}{(2\pi)^6}\int_V\int_V d\mathbf{r}''d\mathbf{r}' <\mu'(\mathbf{r}'')\mu'(\mathbf{r}')> \exp[-iK(x''-x')+ik(\zeta''-\zeta')] \qquad (10) \\
&= \frac{1}{(2\pi)^6}\int_V\int_V d\mathbf{r}d\tilde{\mathbf{r}}R(\mathbf{r})\exp[-iKx+\frac{2ikx\tilde{x}\cos^2\theta/2}{r}+\frac{2iky\tilde{y}\sin^2\theta/2}{r}+\frac{2ikz\tilde{z}}{r}]
\end{aligned}
$$

where the variables in the second equation above have been changed according to $\boldsymbol{x} = \boldsymbol{x}''-\boldsymbol{x}'$ and $\tilde{\boldsymbol{x}} = (\boldsymbol{x}''+\boldsymbol{x}')/2$ and $< \ldots >$ indicates an enmsemble average. In eq. (10) note that we have defines the fluctuating pressure field in terms of the fluctuating index of refraction field by use of the prime notation, namely

$$
\begin{aligned}
&\Gamma'(\mathbf{r}',\tau) \\
&\qquad = \Gamma(\mathbf{r}',\tau) - <\Gamma(\mathbf{r}',\tau)>
\end{aligned}
$$

$$
\begin{aligned}
&\mu'(\mathbf{r}',t) \\
&\qquad = \mu(\mathbf{r}',\tau) - <\mu(\mathbf{r}',\tau)>
\end{aligned}
$$

Using the definition of th index of refraction spectrum as

$R(\boldsymbol{r}''-\boldsymbol{r}') = \int \Phi(\boldsymbol{\kappa})\exp[i\boldsymbol{\kappa}\bullet(\boldsymbol{r}''-\boldsymbol{r}')]d\boldsymbol{\kappa}$ where if Φ is isotropic then $\Phi(\boldsymbol{\kappa}) = \Phi(\kappa)$.

Equation (10) leads to

$$
\Psi = \frac{1}{(2\pi)^3}\int_V d\tilde{\mathbf{r}}\Phi(K+\frac{2k\tilde{x}\cos^2\theta/2}{r},\frac{2k\tilde{y}\sin^2\theta/2}{r},\frac{2k\tilde{z}}{r})\,. \qquad (11)
$$

Note that eq.(11) can be interpetted as a spatial average of the wavenumber spectrum centered at the Bragg wavenumber. At this point no assumption on the nature of the spatial structure of Φ has been made. Let us write the intergand of (11) as

$$\Phi(K+\Delta \mathrm{k}_x, \Delta \mathrm{k}_y, \Delta \mathrm{k}_z) \tag{12}$$

where

$$\Delta \mathrm{k}_x = \frac{2k\tilde{x}\cos^2\theta/2}{r} \tag{13A}$$

$$\Delta \mathrm{k}_y = \frac{2k\tilde{y}\sin^2\theta/2}{r} \tag{13B}$$

$$\Delta \mathrm{k}_z = \frac{2k\tilde{z}}{r} \tag{13C}$$

Eq. (12) can be interpreted as the wavenumber spread on the spectrum Φ due to the effect of the Fresnel phase terms. If these terms can be neglected i.e.

$$\Delta \mathrm{k}_x \approx \Delta \mathrm{k}_y \approx \Delta \mathrm{k}_z \approx 0$$

then use of eq.(12) in (11) yields

$$\Psi = (2\pi)^3 V\, \Phi(K,0,0) = \Phi(K) \tag{14}$$

Thus the Bragg scattering condition is recovered and the received spectrum is proportional to the spectrum of media variability producing the scattering.

Clearly Δk_x, Δk_y, Δk_z in eq.(12) represent the wavenumber shift (and on averaging the wavenumber smearing) of the integral of eq.(11). The criteria for neglecting these quantities is easily establishedSince the spatial extent of the scattering volume is expected to be of order $r\theta_B$, where θ_B is a characteristic beam width angle then the critera for neglecting the firt term (13A) is

$$\Delta \mathrm{k}_x \ll \mathrm{K}$$

$$2\theta_b \cot\theta \cos\theta / 2 << 1 , \qquad (15)$$

where we have estimated the maximum value $\tilde{x}$ in 13 A as

$(\tilde{x})_{max} \approx r\theta_b$

Except for cases of very near forward scattering when

$$\theta_b \approx \theta / 2$$

eq.(15) is well satisified for sufficiently small values of $\theta_{B.}$ This will be the case for the experiment described below. Criteria for establishing the neglect of Δk_y, Δk_z is dependent upon the spatial assymtery of the variability along the three spatial axes.

Let α_y α_z be the degree of anisotropy of Φ in the "y" and "z" direction relative to the "x" axis, and let us assume that the "x" axis of the Bragg direction is aligned in the direction of the smallest principal axis of symmetry, then both

$a_y \le 1$ and $a_z \le 1$

it follows that

$$\Phi(K, k_y, k_z) \approx \Phi(K, 0, 0)$$
if
$$k_y << \alpha_y K \qquad \& \qquad (16A)$$

$$k_z << \alpha_z K \qquad (16B)$$

and eq. (14) follows...

which recovers the Bragg scattering condition for the wave number spectrum. To establish the criteria for this, substitution of 13 A, B into 16A, B yields

$$2\theta_{\mathrm{b}} \sin\theta/2 << \alpha_{\mathrm{y}} \quad (17A)$$

$$\theta_{\mathrm{b}} << 2\alpha_{\mathrm{z}} \sin\theta/2 \quad (17B)$$

For the scattering angles used in the experiment of the manuscript these conditions are approximate by

$$\theta_{\mathrm{b}} << \alpha_{\mathrm{z,y}} \quad (18)$$

Thus if the degree of anisotropy parameter $\alpha_{\mathrm{z,y}}$ is not smaller than θ_{b} the Bragg scattering condition can be applied. This result also holds for the two-dimensional scattering case associated with (two-dimensional) turbulent like variability.

To obtain the acoustical estimate of the three-dimensional case we substitute γ_3 from Eq. (6) into Eq. (14) using the defintion of the wavenumber spectrum to obtain for the acoustically derived three- dimensional wave number estimate of index of refraction, namely

$$\Psi_{3D}(K) = \frac{1}{V(2\pi)^3} |h(\omega)|^2 (2\pi r)^2 (2\sin(\theta/2))^4 K^{-4} \quad (19)$$

where V is the scattering volume defined by the beam pattern intersection. Analogously we can obtain the estimate for the two-dimensional scattering case, which can be shown to be

$$\Psi_{2D}(K) = \frac{1}{A(2\pi)^2} |h(\omega)|^2 (2\pi r)(2\sin(\theta/2))^3 K^{-3} \qquad (20)$$

where A is the area of the two-dimensional scattering field. For the case of the baffled pistons used in the experiments defined below, we take $V = L\,A$ with $L = 5.4\, r/(ka)$, the vertical extent of the scattering volume defined to the 10 dB down point, where r is the radius to the scattering volume and a is the transducer radius. The ratio of the three-dimensional estimate to the two-dimensional estimate is then found to be

$$\frac{\Psi_{3D}(K)}{\Psi_{2D}(K)} = 0.2a \qquad (21)$$

Thus the three-dimensional estimate will scale with Bragg wave number the same as the two- dimensional estimate and differ by a factor of order a. Equation (21) will have a very important consequence in comparing the acoustically estimated index of refraction spectrum of the plume with that derived from a direct measurement of index of refraction (temperature). For equations (19), (20) and (21) to be valid condition of eq.(18) must be valid.

III. EXPERIMENTAL SETUP

The laboratory consists of a 2m x 2m x 1.2m polypropylene tank filled with fresh tap water and filtered down to 5 microns. Glass windows are located on each side of the tank allowing use of a laser shadowgraph system to image the thermal plume and qualitatively monitor its condition. The heating element used to generate the plume is inside a cylindrical stainless steel housing having outside diameter of 1cm, 3.8 cm high, elevated 6 cm above a bottom mounting plate and located 25 cm below the horizontal scattering plane. The input power to the heating element is controlled with a Variac with a maximum output power of 50 watts. The acoustic volume is centered on the plume axis by using a ring assembly system to mount the transducers. The assembly consists of a plate having a 50-cm diameter inner radius around which transducers are placed in 10-degree increments. For this work, three pairs of reciprocal source-receiver transducers were used, each having 10 dB down points at 425 and 725 kHz. The transducers have a crystal diameter of 1.5" and are well modeled by baffled pistons.

The transmitted acoustic pulses are generated using an arbitrary waveform generator consisting of a single cycle pulse with center frequency at 500 kHz and amplified with 2 kW amplifiers. Each of the three source transducers are stimulated separately with a 1 ms time lag between transmissions, thus sufficiently reducing tank reverberation yet still allowing interrogation of essentially the same scattering field. Since the nominal plume velocity in the vertical is 3 cm/s, typical vertical displacement over 2.0 ms (the time delay from the stimulus of transmitter one until transmitter three is fired) is approximately 0.06 mm, which has a negligible effect on the results. Each receiving channel is range gated such that for each ping the acoustic scatter from the plume is centered in the 512 samples that are digitized at 5 MHz. The system repetition rate is 67 Hz and 2,048 waveforms per channel are collected. The high repetition rate allows for estimating one component of the turbulent velocity using a coherent acoustic Doppler technique and is a subject of continuing research. Incident acoustic signals at the scattering volume are measured by placing the receiving transducer directly facing the source transducer across the ring.

A Seabird SBE 3-01/F fine structure temperature probe is also placed on the plume axis 7.5 cm above the horizontal acoustic scattering plane. The probe has a measured e-folding time of 67 msec. Temperature time series measurements are sampled at 60 Hz and made simultaneously with the acoustic scattering measurements. Because of the inhomogeneous nature of the plume turbulence, the spectral estimates made 7 cm above the scattering volume differed from the estimate made at the center of the volume. Thus, temperature measurements are also made both before and immediately after the acoustic measurements, with the temperature probe however, moved to the center of the scattering volume.

IV. RESULTS

The data collected for the measurements made in the common Bragg scattering configuration (Figure 2) direction have scattering angles of 80, 120, and 160-degrees measured, respectively, from the forward direction for each of the three source-receiver pairs used. The fluctuations about the mean are used for the analysis and are determined by removing the mean waveform for each channel from each waveform of the respective channel. That is, let the n^{th} ping received on the i^{th} channel be represented by $p_i(t_j, \tau_n)$; the variable "t_j" represents the time of the sampled points per ping, while the τ_n is the repition or sampling time interval. For this manuscript , $i=1, 2$, or $3; j=1,...,512$; and $n=1,...,2048$. The "t_j", can be converted to a spatial coordinate through the relation $x_{i,j} = ct_j /(2 \sin\theta_i / 2)$, while the Bragg wave numbers for each channel are determined by $K_i = 2k \sin(\theta_i / 2)$. The mean waveform for the i^{th} channel is then expressed by $\overline{p}_i(t_j) = < p_i(t_j, \tau_n) >$, so that the fluctuations of the i^{th} channel are represented by $p'_i(t_j, \tau_n) = p_i(t_j, \tau_n) - \overline{p}_i(t_j)$. The angle brackets represent an ensemble average over the N= 2048 waveforms taken for each experiment. It is assumed that μ does not vary over time scales t_n which are of order 10’s of microseconds. Note that the Fourier transform of p’ on t is Γ’.

A. Unstable Plume

For the purposes of this paper , the buoyant thermal plume has three different dynamical regimes denoted by laminar, unstable, and turbulent flows. These regimes are obtained by varying the input power to the heating element. Nominally, the plume remains laminar at the height of the scattering plane for an input power level less than 25

W. An unstable plume exists for input power levels between 25-35 W and turbulent for power levels greater than 35 W. It should be noted however, that these power levels might shift by as much as 50% in either direction over the period of hours. For the data presented in this section, the unstable plume is generated with an input power level of 35 W. The unstable plume for this case is characterized by a slow oscillation in the horizontal plane giving rise to a sinusoidal-like plume structure in the vertical direction with a nominal vertical wavelength of 5 cm.

Consider the three-dimensional spectrum of the index of refraction field which we will estimate with Ψ_{3D}, using Eq. (19) in the common Bragg scattering configuration. Figure 3(a) shows Ψ_{3D} for an unstable plume at 22.575 seconds into a 30.72 second measurement run. The dashed, dotted, and solid lines are the contributions to Ψ_{3D} from the 80, 120 and 160-degree scattering angle data sets, respectively. An estimate of Ψ_{2D}, using Eq. (20), will yield the exactly same figure but differing by the factor 0.2a as discussed and shown by Eq. (21). (Note that the units would be different also, i.e. $(\mathrm{rad/m})^{-2}$ instead of $(\mathrm{rad/m})^{-3}$). The strong degree of overlap suggests that the Bragg scattering condition is well satisfied but at this stage of analysis we cannot say if scattering from the unstable plume is a process which is more three-dimensional than two-dimensional. We will return to this point later.

Returning to the very good agreement in the spectral magnitude in the regions of overlapping Bragg wave numbers, these occur between channels one and two for wave numbers from 2000 to 3000 rad/m and for channels two and three from 3500 to 5500 rad/m. Noteworthy is the presence of deep nulls in the wave number spectrum, indicative of sharp temperature gradients in the thermal plume.

A contour plot of $\Psi_{3D}(K,\tau)$ versus Bragg wave number and time is shown in Fig. 3(b). All three channels of data are combined into one matrix, sorted by wave number component, and interpolated over an evenly spaced set of wave numbers corresponding to the range of Bragg wave numbers available from the scattering measurement. Note the smooth variation in the magnitude of the spectrum as a function of wave number. This is seen throughout the 30-second time series measurement and indicative of the success of the common Bragg wave number comparisons. The periodicity in the spectrum is evident as expected for an unstable plume. It is interesting to examine the slope of constant contours in the spectrum, indicative of constant null spacing in the spectrum throughout the time series. There results a typical value of $\Delta K_x / \Delta\tau = 1250$ rad/(ms) to 2500 rad/(ms). Laser shadowgraph observations suggest in the vertical direction ($\lambda_z = 5\,cm,\ T = 1.7\,sec$) a value of $\Delta K_z / T = 74$ rad/(ms). The ratio of these two quantities results in an estimate for the length scale of horizontal variability between $l_x = l_z / 17 = 2.9\,mm$ and $l_x = l_z / 34 = 1.5\,mm$.

If the temperature profile is modeled as a cylinder, then the solution for the scattered pressure will vary as $J_1(Kb)$, where J_1 is the Bessel function of order one, and b is the radius of the cylindrical plume profile. Except for the first pair, the spacing between the first few zeros of $J_1(x)$ is constant ($\Delta x_i =$3.83, 3.19, 3.15, 3.15, 3.15). Given an estimate for b of 2.5 mm from above, the first Δx can be safely ignored since that null occurs at K = 1530 rad/m, which is outside of the available Bragg wave number bandwidth. From Figs. 3(a) and (b), an estimate of the plume width is found by measuring the null spacing in wave number and calculating $b = \Delta x_3 / \Delta K$. Typical

ΔK values between nulls range from 1,200 to 2,000 rad/m; these correspond to estimates of b between 1.6 and 2.6 mm. These estimates are in close agreement the value of 2.4 mm associated the laminar plume described in an earlier paper.[1]

Figure 3(c) is the mean three-dimensional wave number spectrum of the index of refraction fluctuations for the unstable plume. The spectrum includes data from all three channels and indicates the good agreement between the regions of overlapping Bragg wave numbers. For the unstable plume, the spectrum in the given wave number range falls off rapidly at a power law of K^{-9}.

The wave number frequency spectrum defined by

$$\Psi_{3D}(K,\omega) = \frac{(2\pi)^4}{TV}\left\langle \gamma'_3(K,\omega)\gamma'^*_3(K,\omega)\right\rangle \tag{22}$$

where $\gamma'_3(K,\omega)$ is the Fourier transform in time of $\gamma'(K,\tau)$ from Eq. (6), K is the Bragg wave number V the scattering volume and T the total time over which the data was sampled. Figure 4 clearly shows the periodicity of the unstable plume over the entire bandwidth of wave numbers with the peak magnitude occurring between 3 and 4 rad/s and a second local maximum around 7 rad/s. From the relationship of $\omega = vk_z$ and using 3.5 rad/s for ω and 3 cm/s for v the value for the wave length of the oscillation in the vertical direction is 5.4 cm. This is in close agreement with laser shadowgraph observations.

To compare the acoustically derived results with those from the temperature probe we need to convert the acoustic wave number frequency spectrum to a frequency spectrum since the temperature probe is limited to a point time series measurement. An acoustically estimated one-dimensional frequency spectrum $\Psi(\omega)$ is formally related to the three-dimensional wave number/frequency spectrum by

$$\Psi_{3D}(\omega) = \int d^3k'\Psi_{3D}(k',\omega) \tag{23)}$$

and to a two-dimensional wave number/frequency spectrum by

$$\Psi_{2D}(\omega) = \int d^2k'\Psi_{2D}(k',\omega)\,. \tag{24}$$

Since we only have wave number information in one direction we need to make an assumption about the statistical nature of the variability in the other two spatial directions. The assumption that we make (under the critera developed in section II) is that of spatial near isotropy over the Bragg wave number range (which corresponds to mm range spatial scales), which then results in

$$\Psi_{3D}(\omega) = (4\pi)\int_{\mathrm{k}_1}^{\mathrm{k}_u} dk'k'^2\,\Psi_{3D}(k',\omega) \tag{25}$$

where k_l and k_u are upper and lower limits of integration defined by the finite bandwidth used in the experiment. If a two-dimensional scattering process were assumed and Eq. (24) utilized again with the same assumption then the frequency spectrum would be

$$\Psi_{2D}(\omega) = (2\pi)\int_{\mathrm{k}_1}^{\mathrm{k}_u} dk'k'\Psi_{2D}(k',\omega) \,. \qquad (26)(26)$$

Using Eq. (21) such that

$$\Psi_{3D}(K) = 0.2a\Psi_{2D}(K) \qquad (27)$$

results in

$$\begin{aligned}\Psi_{3D}(\omega) &\approx K^* a\Psi_{2D}'(\omega) \\ &>> \Psi'(\omega)\end{aligned} \qquad (28)$$

where K^* is a characteristic wave number whose value is expected to be between k_l and k_u, and depends on the details of the integration. Thus if one uses a three-dimensional estimate, the estimate $\Psi_{3D}(\omega)$ will be much larger than that from a two-dimensional estimate $\Psi_{2D}'(\omega)$ by the factor $K^* a$. This can be understood by the fact that a two-dimensional scattering process is coherent in the third dimension, which in our case would be along the axis of the plume. A field of three-dimensional scattering would have many independent scatters distributed along the axis of symmetry. The ratio of the intensities of the two cases would be expected to vary as the number of independent samples and would be of order K^*a, which agrees with the above analysis. Thus a three dimensional scattering process would require a higher level of variability in general than a two dimensional one since the two dimensional process is by defintion coherent in the third dimension. Equation (25) (and/or Eq. 26) can be used for comparison with the one-dimensional spectrum estimated from the temperature probe measurements. The spectrum estimated from the temperature probe is corrected for the probe response time and converted to index of refraction. The one-dimensional spectral comparisons are shown in Fig. 5. The acoustically and mechanically (temperature probe) estimated spectra both contain peak values at 3.7 rad/s and 8 rad/s. The magnitude of the temperature probe derived spectra is about an order of magnitude larger than the acoustically derived one. This is not unexpected due to the acoustic estimate resulting from a band limited measurement and the assumption of isotropy for the unstable plume, which may not be valid. However application of the isotropy assumption to scales of order the Bragg wave number (~mm wavelengths) might be expected to be a relatively good approximation since some level of small scale turbulence is present in the unstable plume case and turbulence at these scales is expected to be isotropic.[26] It is also interesting to note that it can be shown that the assumption of isotropy, analogous to the argument presented in contrasting two- and three-dimensional scattering, results in an upper bound estimate of the frequency spectrum $\Psi(\omega)$. That the acoustic estimate is below that from the temperature probe suggests that the explanation for the difference in these curves is that there are contributions to plume temporal variability from a range of spatial scales larger than that which can be resolved by the common Bragg wave number technique and would correspond to wave numbers (frequencies) below the resolution of the transducers being used.

B. Turbulent Plume

Analysis similar to the unstable plume is conducted for the acoustic scattering from the turbulent plume. Figure 6(a) is a record of the three-dimensional wave number spectrum of the index of refraction fluctuations, $\Psi_{3D}(K,\tau)$, of the turbulent plume at τ = 22.575 seconds. This figure shows the results of common Bragg wave number configuration for the three channel multi-static measurement. In the regions where there are common Bragg wave numbers between adjacent channels, the magnitude of the wave number spectrum between the channels is in good agreement but do contain regions of divergence between the channels. The pronounced structure that is present in the spectrum is typical for scattering from the turbulent plume. The nulls in the wave number spectrum can be as much as 30-40 dB below the mean spectral value. It should be noted that ping-to-ping records of the spectrum tend to remain highly coherent. The deep nulls indicate that the turbulence has a low Reynolds' number and thus is not fully developed. Figure 6(b) is a contour plot of the entire time series wave number spectrum of the index of refraction fluctuations for the turbulent plume. One of the first noticeable features in this figure, especially when compared to Fig. 3(b) of the unstable plume, is the absence of the periodic nature of the spectrum. As expected, the surface plot appears almost random in comparison, although there is still noticeable structure at smaller scales. Figure 6(b) also indicates the smoothly varying continuity over the entire bandwidth of wave numbers for the three-channel multi-static measurement. It is evident that the spectrum rolls off with increasing wave number, as expected for turbulence. The mean value of the three-dimensional wave number spectrum is given in Fig. 6(c). The solid black line represents the –11/3 power law expected for classical turbulence and is given for comparison with the acoustically estimated wave number spectrum.[26] The acoustically estimated spectrum rolls off slightly faster than classical turbulence, not an unexpected result given the structure of the spectrum in Fig. 6(b) showing the nulls and the low value of the Reynolds number.

Figure 7 is a contour plot of the wave number/frequency spectrum for the data represented in Fig. 6(b), namely $\Psi_{3D}(K,\omega)$. Smoothly varying structure is noticeable throughout the entire wave number/frequency spectrum. There is also a perceptible trend in the spectrum such that the largest spectral values tend to coalesce to a confined region and form an apparent wave number-frequency dependence. A closer examination of these trends indicates that the slopes along the edges of the coalesced regions have nominal values between 0.5 and 3 cm/sec. These values are well within the velocities expected for the turbulent plume, with maximum value of 3 cm/sec associated with the mean vertical advection, and the lower values likely due to the turbulent eddies within the plume. Although not a part of the present work, coherent Doppler estimates from the acoustic scatter suggest the broadband averaged rms value for the fluid velocity of the present data set yield approximately 2 mm/sec.

The one-dimensional spectral comparison of the index of refraction fluctuations in the turbulent plume between the acoustical and temperature probe estimates is given in Fig. 8. These calculations were performed in the same manner as described for the unstable case. Between 20 and 50 rad/s there is remarkably good agreement between the two types of measurements, both in terms of magnitude and roll-off. Note that the acoustically estimated spectra levels off below 30 rad/sec. Using an advection velocity for the plume of 1.5 cm/sec yields a cutoff wave number of 2000 rad/m which is of order the lowest estimated wave number of 2300 rad/m which can be estimated from the Bragg scattering geometry employed. If the acoustic signal were extended to lower wave numbers by either incorporating smaller scattering angles in the common Bragg wave number configuration or by using lower acoustic frequencies, the lower frequency part of the index of refraction (temperature) field as measured by the mechanical probe would be able to be estimated.

V. CONCLUSIONS

The issue addressed in this paper is the verification of application of far-field weak scattering theory to the case of acoustic scattering from thermally generated sound speed variations in water. The motivation for the usage of high frequency broadbandwidth multi-static acoustic scatter from medium variability is a consequence resulting from a prediction of far-field weak scattering theory, namely that the spectral magnitudes must be equivalent for the acoustic estimates from multi-static data taken coincidently in space and time for regions of overlapping Bragg wave numbers.

It has been shown that the multi-static acoustic scatter from both unstable and turbulent plumes validates the prediction made by the far-field weak scattering theory when ensemble averaging is employed to calculate the wavenumber spectrum. This is validated by the overlap of the spectral estimates shown in Figures 3Aand 6A for the unstable and buoyant plumes, respectively. The only *in situ* verification possible to date for our results has been to compare a reduced form of the spectral results, using either Eq. (23) or (24), to that obtained from a point temperature sensor. Note that the he common Bragg wave number technique allows both a space and time measurement of the scattering field whereas the temperature measurement only gives the time variability at a point.

We have also shown that nless some *a priori* information is known about the nature of the dimensionality of the scattering a two-dimensional inversion will yield the same shape wave number spectrum as a three-dimensional inversion. See Eqs. (6), (27), (28) and the discussion associated with those equations. Thethree-dimensional inversion was appears to better describe the structure of both the unstable plume and turbulent cases. For the turbulent case, the acoustically estimated one dimensional frequency spectrum was in very good agreement with that obtained by the temprature sensor over frequency ranges corresponding to the Bragg wavenumber, Fig. 8.

The actual criterion of maximum anisotory allowed to use far field Bragg scattering is not overly constraining. To see this, note as was discussed in section II C the degree of anisotropy that can be tolerated given by eq. (17B) is

$$\theta_b \quad << 2\alpha_z \sin\theta / 2 \approx \alpha_z \qquad (29)$$

. <u>For our experiment from Table 1</u>

$\theta_b \approx .02 \quad to \quad .03$

Thus the maximum degree of anisotropy which could be tolerated is expected to be of order .1 and we conclude from our experimental results because of the common overlap of the spectral estimates shown Figures 3A and 6A that both the unstable and turbulent plume satisfies this criterion and do not have anisotropy values of θ_b less than of order .1.

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APPENDIX A

The relative compressibility and density of a region of medium variability are given by $\gamma_\kappa = \dfrac{\kappa - \kappa_o}{\kappa_o}$, and $\gamma_\rho = \dfrac{\rho - \rho_o}{\rho}$ where the compressibility $\kappa = 1/\rho c^2$ where ρ and c are the anomalous density and sound speed, respectively.[21] Expressing these latter variables as the sum of the ambient value and its fluctuation, (i.e., $\rho = \rho_o + \rho'$ and $c = c_o + c'$) and since the fluctuations are expected to be small, the relative compressibility is approximated to first order by

$$\gamma_\kappa \cong -(2c'/c_o + \rho'/\rho_o). \tag{A1}$$

The relative density is approximated as

$$\gamma_\rho \cong \frac{\rho'}{\rho_o}. \tag{A2}$$

Given a nominal ambient water temperature of 20 ^{o}C and a maximum temperature difference from ambient of 5 ^{o}C, the terms in Eqs. (A1) and (A2) are readily determined[27] to be $2c'/c_o =$ 0.02, and $\rho'/\rho =$ -0.001. Thus it is observed that thermally induced sound speed dominates density for the parameters given this manuscript. It should also be noted that effects due to salinity variability, while the subject of current investigation, are not pertinent in the present case of scattering from thermal driven variability in fresh water.

The relevant term for the relative fluid velocity is given by $2u'/c_o$.[1,20] For the case of the buoyant plume having a nominal vertical speed of 3 cm/s and used as the upper limit for the fluctuation of fluid velocity, $2u'/c_o = 0.00004$ and can be safely ignored compared to the thermally driven sound speed.

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FIGURE CAPTIONS

Figure 1. Bragg wave number vector scattering geometry. Notice the x-axis is defined as the bisector of the source to volume and volume to receiver cords. The scattering angle θ is measured from the forward direction.

Figure 2. Common Bragg direction multi-static scattering configuration. All source-receiver pairs have a common Bragg wave number vector direction and thus share the same bisecting x-axis.

Figure 3a. Common Bragg wave number spectrum of index of refraction fluctuations for the unstable plume at t = 22.575 seconds. The spectrum is formed by combining data from three separate receiver channels corresponding to scattering angles of 80-degrees (dashed), 120-degrees (dotted) , 160-degrees (solid). The black dots are an interpolation of the three curves over the entire range.

Figure 3b. Time Series of the interpolated common Bragg wave number spectrum of index of refraction fluctuations (black dots of Fig. 3a) for the unstable plume case.

Figure 3c. Temporal mean wave number spectrum calculated from Figure 3b, the case of the unstable plume.

Figure 4. Wave number/Frequency spectrum of index of refraction fluctuations for

the unstable plume case. Same data as that used in Figure 3b.

Figure 5. One-dimensional spectrum of temperature for the unstable plume case. The dashed curve is calculated by using a thermometer with a correction for the high frequency response included. The solid curve is the acoustically derived one-dimensional spectrum, calculated by integrating Figure 4 over "three-dimensional" wave number space assuming isotropy.

Figure 6a. Common Bragg wave number spectrum of index of refraction fluctuations for the turbulent plume at t = 22.575 seconds. It is formed by combining data from three separate receiver channels corresponding to scattering angles of 80-degrees (dashed), 120-degrees (dotted), 160-degrees (solid). The black dots are an interpolation of the three curves over the entire range.

Figure 6b. Time Series of the interpolated common Bragg wave number spectrum of index of refraction fluctuations for the turbulent plume. It is formed by combining data from three separate receiver channels corresponding to scattering angles of 80-degrees (Green), 120-degrees (Blue) , 160-degrees (Red).

Figure 6c. Temporal mean wave number spectrum calculated from Figure 6b.
The dashed line of -11/3 is the power law expected for an isotropic turbulent spectrum.

Figure 7. Wave number /Frequency spectrum of index of refraction fluctuations from a turbulent plume. Same data as that used in Figure 6b.

Figure 8. One-dimensional spectrum of temperature for the turbulent plume case. The dashed curve is calculated by using a thermometer with a correction for the high frequency response included. The solid curve is the acoustically derived one-dimensional spectrum, calculated by integrating Figure 7 over "three-dimensional" wave number space assuming isotropy.