MAR 670 Ocean Data Analysis
Due XXXXXXX April 2008

Exercize 7. Massachusetts Internal Tides

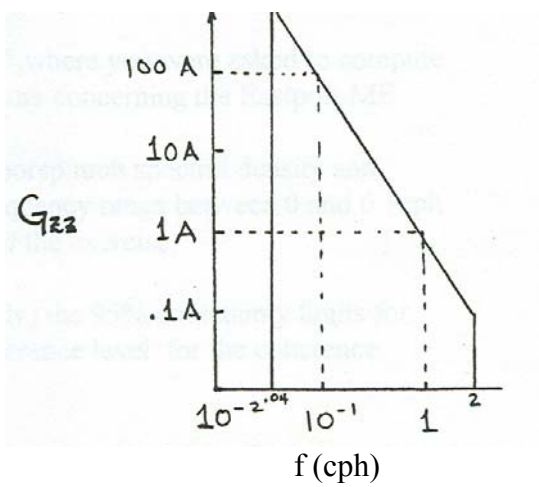
<u>Given: Spectra of Idealized Internal Waves</u>

Density stratification of the ocean water column enables the vertical displacement of isopycnals (i.e. surfaces of constant density). In theory, the <u>maximum</u> allowable frequency for these isopycnal displacement oscillations is the local buoyancy frequency N (where N^2 α stratification strength); the <u>minimum</u> is the <u>local</u> inertial frequency f_I (which varies with latitude).

The idealized autospectrum of a typical mid-ocean isopycnal displacement time series z(t) due to internal gravity waves in the has been approximated by Garrett and Munk (1972) as

$$G_{zz}(f) = A/f^2 \quad \text{for} \quad f_I = 0.04\text{cph} < f < N = 2\text{cph}$$
$$= 0 \quad \text{for} \quad f > 2\text{cph} \text{ and } f < 0.04\text{cph}$$

where the value of A depends on the ocean depth at which the observations are made; typically **A = 1 (m^2-cph)** at a depth of 1000m.



(a) What are the units of $G_{zz}(f)$ in the equation above?

$$[G_{zz}(f)] = [A] / [f^2] = m^2\text{-cph} / \text{cph}^2 = m^2/\text{cph}$$

(b) What is the variance of the idealized mid-ocean internal waves (IW) within a frequency band $\Delta f = 0.007142857$cph that brackets the M_2 semidiurnal frequency of 0.0805cph?

upper frequency limit $f_u = 0.0805 + \Delta f/2 = 0.0805 + 0.003571428 =$ **0.0841 cph;**
center frequency f_c **= 0.0805 cph;**
lower frequency limit $f_l = 0.0805 - \Delta f/2 = 0.0805 - 0.003571428 =$ **0.0769 cph;**

$$\textbf{Variance } M_2 = \int_{f_l}^{f_u} G_{zz}(f)\, df = -A f^{-1} \Big|_{0.0769\text{ cph}}^{0.0841\text{ cph}} = \underline{1.113\ m^2}$$

What are the corresponding standard deviation ?

$$\textbf{Std. Dev.} = \sqrt{(\textbf{Variance } M_2)} = \underline{1.055\ m}$$

and the <u>equivalent amplitude</u> of an internal wave represented by that variance.

For a sine wave with, zero mean value, amplitude **a,** and phase $2\pi f_c t = \alpha$, the

$$\text{Variance } M_2 = \underline{\text{mean}} \text{ square} = (2\pi)^{-1} \int_{-\pi}^{\pi} a^2 \sin^2\alpha\, d\alpha = (2\pi)^{-1} a^2 \int_{-\pi}^{\pi} (1-\cos 2\alpha)/2\, d\alpha = a^2/2$$

Therefore $a_{equ} = \sqrt{(2 \times \text{Variance } M_2)} = \text{Std. Dev. } M_2 \times \sqrt{2} = \underline{1.492\ m}$

(c) What is the <u>total</u> IW variance (and corresponding standard deviation) in the range $0.056\text{cph} < f < 0.500\text{cph}$?

$$\textbf{Wideband } G_{zz}(f) \textbf{ Variance} = \int G_{zz}(f)\, df = -A f^{-1} \Big|_{0.50\text{ cph}}^{0.056\text{ cph}} = \underline{15.857\ m^2}$$

$$\textbf{Std. Dev.} = \sqrt{(\textbf{Wideband } G_{zz}(f) \textbf{ Variance})} = \underline{3.982\ m}$$

What is the percentage of the total variance that is due to the M_2 semidiurnal frequency band variability?

$$\% = 1.113\ m^2 / 15.857\ m^2 = \underline{7.02}$$

Problem 1 Statistical Analysis of Observed Internal Waves

Hourly-averaged time series of pairs of temperature/conductivity measurements were made at 4, 10, 25, 45 and 60m depths, respectively, in 87 m of water from a mooring deployed in Stellwagen Basin (Massachusetts Bay; f_I = 0.056cph; 1cph < N < 30cph) at 42° 21.2'N, 70° 24.3'W from July 8 1990 through 10 August 1990. These measurements were used to compute a time series of density anomaly (i.e., sigma-theta) from which a set of isopycnal displacement series were estimated via linear interpolation.

Here we consider 3 time series of 856 hourly samples of isopycnal displacement for sigma theta = 23.70; 25.30; and 25.55 (files called i237.sum, i253.sum, and i2555.sum respectively) are in */hosts/XXXXXXXXXXXXX/exam_I.dir*. It is clear in *Figure 1* that internal waves at tidal frequencies – internal tides - dominate the 2 deeper records tides. But there is other variability also.

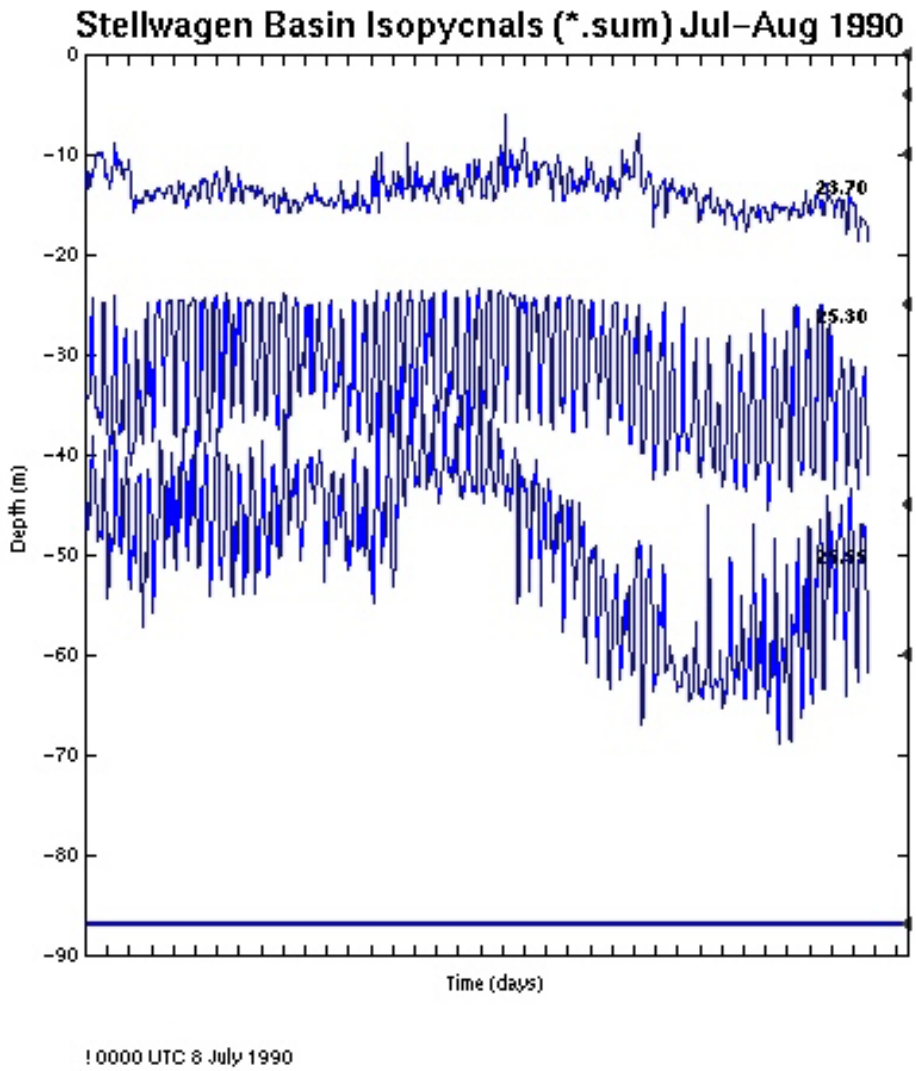


Figure 1. The *.sum isopycnal depth records.

The bulk statistics of the 3 *.sum series on a 0 to -100m depth versus time are:

name	maximum (m)	minimum(m)	mean (m)	std dev(m)	var(m^2)
i237.sum	-.595848E+01	-.186592E+02	-.135789E+02	1.72931	2.9905
i253.sum	-.234051E+02	-.453628E+02	-.315865E+02	5.62363	31.6252
i2555.sum	-.308666E+02	-.687543E+02	-.493001E+02	7.98109	63.6978

(a) In an effort to focus on the internal tidal wave frequency band, you are asked to apply a high pass filter to these series. Use **l_filter** [36-term, lanczos high pass filter with a cutoff frequency = 0.056 Nyquists] on the three isopycnal series to *produce the corresponding *.hs series.*

(1) What is the dimensional cutoff frequency of the high pass filter?

(2) Why is it necessary to remove the low frequency variability?

(b) Compute the bulk statistics (**l_lstats**) and plot *the *.hs series* using the same format as in (a).

How much and what percentage of the original variance was due to the low frequency variability?

(c) Compute (**l_specter** with a fundamental frequency band of 0.007142857cph) to compute the autospectrum of the filtered 25.30 isopycnal displacement series.

(1) Plot the autospectrum on log-log axes of dimensional spectral energy density and frequency...complete with the appropriate 95% uncertainty limits for the relevant degrees of freedom (DOF).

(2) Assuming that f = 0.007142857cph is in the center of the first harmonic, at what harmonics (and frequencies) do you find <u>statistically significant</u> peaks in the spectrum?

(3) At what frequency band do you find most of the variance?

(d) Use cross-correlation function analysis to briefly look at the structure of these isopycnal displacement time series.

(1) Specifically, compute the trio of cross-correlations (50 lags) of both the 3 *.hs* series using **l_correl** (option 3 so you can obtain the "long lag"

normalized form). Plot the cross-correlations, including the 95% zero correlation uncertainty levels.

(3) Describe any statistically significant (at the 95% confidence level) cross-correlations in terms of magnitude and time delays.

Problem 2 Comparison of Theoretical and Observed Internal Wave Variability

The analysis results of Problem 2 show that the observed internal waves in Massachusetts Bay have a strong tidal frequency component that the mid-ocean internal waves do not. In order to make a meaningful comparison between the 2 regimes, we must "remove" the internal tides from the Stellwagen basin records.

(a) Remove the tides by applying an *optimal filter* – tidal harmonic analysis **(l_tiharc98)** in this case- to the *i253.hs* series.

(b) Compute bulk statistics and plot the detided, filtered series**.**

What, if any, visual differences are there?
If there are visual differences, explain using the low pass filter response if appropriate.

(c) Compute the autospectra (**l_specter**) for the series and plot on the same format as above.

(d) Compare the Stellwagen Basin and mid-ocean internal wave variance in the frequency band containing f = 0.0805cph.in terms of the (a) variance, (b) standard deviation, (c) equivalent amplitude (d) and percentage of the 0.056cph < f < 0.500cph frequency band energy.

Problem 3. SISO Transfer Function: **Internal Tidal Forcing**

The interaction of tidal currents and bathymetry are known to generate internal tidal waves. Above you have analyzed the internal tidal signals in Stellwagen Basin above. Here we ask you to compute the Single Input/Single Output (SISO) transfer function and the associated noise between a relevant current measurement and the output"signal". For these purposes an hourly-averaged time series of current in 20 m of water from a mooring deployed on Stellwagen Bank is provided in **/hosts/XXXXXXXXXXXXXX/exercize7.dir.** Use the fundamental frequency bandwidth from Problem 1 above and estimate uncertainty limits.