Homework 7: Waves

7-1. TYPES OF WAVES

An ocean surface wave is a water surface oscillation – or an alternating rise and fall of the water surface. Waves are generated when a disturbing force, such as wind, moon- and sun-related gravitational force, or sea floor earthquake faulting, distorts the sea surface from it nominally flat condition. The distorted sea surface (the crest in Figure 7-1) has potential energy (PE - energy of position), but is unstable to gravity. The gravitational restoring force immediately begins to convert this PE into vertical motion or kinetic energy (KE - energy of motion) that returns the sea surface towards its originally flat configuration. However, the downward moving sea surface overshoots its original position until it reverses its motion in the trough (due to buoyant forces) - thus resulting in the oscillating wave motion.

Waves are either progressive waves that move or propagate away from their generation site; or standing waves that do not propagate; or a combination of both. The real ocean surface is complicated, so they are best understood by considering a simple waveform in Figure 7-1.

7-2. WAVE CHARACTERISTICS

Our simple wave (Figure 7-1) is defined by its wave length, wave height, wave amplitude, wave period, and wave frequency:

- The wave length \( (L) \) is the length of one complete wave form, as measured from wave crest to adjacent wave crest (or trough to trough).

- The wave height \( (H) \) is the vertical distance between the wave crest and wave trough.

- The wave amplitude \( (A) \) is the vertical distance between of the undisturbed water level and either the crest or trough; and is equal to one-half the wave height (i.e., \( A = 1/2 \ H \)).
• The **wave period** (T) *for a progressive wave* is the time it takes for one full wave to pass a fixed point. The wave period *for a standing wave* is the time for one complete oscillation or cycle of the water surface.

• The **wave frequency** (F) is the inverse of the wave period (F=1/T) or the *number of wave cycles per unit of time*.

• The **wave speed** (C\(_p\)) of a progressive wave is the rate at which a *wave form* to propagates across the sea surface; specifically, the ratio distance/time or C\(_p\) = L/T.

### 7-3. PROGRESSIVE WAVE MOTION IN DEEP AND SHALLOW WATER

Progressive waves in deep water or **deep water waves** propagate away from their generation site carrying wave energy. However, the water parcels involved in the wave motion do not move with the wave, but rather they move in near-circular trajectories or **orbits** beneath the passing wave as shown in Figure 7-2. In a progressive wave, water parcels move in the propagation direction as the crest of the wave passes. The **diameters of the water parcel orbits** decrease from the surface one that is equal to the wave height to virtually zero at the **wave base**, which at a depth equal to ½ wave length. By definition, deep water waves propagate in water that is deeper than the wave base. Thus, a scuba diver swimming at depths deeper than the wave base are unaware that waves are propagating overhead.

![Figure 7-2. Deep water wave motion. In “deep water”, where the ocean depth is greater than ½ the wavelength or D/L > 0.5, progressive wave water parcels move in circular trajectories or orbits; orbits that decrease in diameter from H at the surface to near zero at the wave base.](image)

The speed of a deep water wave (C\(_p\)) depends on wave period (easier to measure) and/or the wave length and is given by:

\[
C_p (\text{m/s}) = 1.56T (\text{sec}) = 1.25 \sqrt{L} (\text{m})
\]

In the open ocean, where D ~ 3km, most waves are deep water waves (tsunamis and tides are important exceptions). However, deep ocean waves do eventually propagate into
water depths (D) that less than half their wave length (D = 1/2 L). At that point, the waves are said to "feel the bottom". As the water shallows or shoals, wave lengths decrease and wave heights increase as waves are "squeezed" together. Also, water parcel orbits become ellipses (Figure 7-3), which become flatter with depth to the point near the bottom where they become simple back-and-forth motion. In water depths less than 1/20 L, the waves become shallow water waves (Figure 7-4) at which point their speed depends on water depth D according to:

$$C_p \text{ (m/s)} = 3.1 \sqrt{D \text{ (m)}}$$

**Figure 7-3.** Deep water waves propagating into “shallow water”. Note how the waves “feel the bottom” and water parcel orbits are squashed as the wave propagates into even shallower water.

(b) SHALLOW-WATER WAVE

**Figure 7-4.** Shallow water wave motion. In “shallow water”, where the ocean depth is less than 1/20 the wavelength or D/L > 0.05, progressive wave water parcels move in elliptical orbits; orbit widths that are constant with depth all the way to the bottom.

As the waves approach the beach, they get steeper. The stability of a wave front is related to its steepness S - the ratio of wave height to wave length (S = H/L). When H/L becomes greater than 1/7, the wave face becomes unstable and breaks (Figure 7-5).
7-4. WIND-GENERATED PROGRESSIVE WAVES

Most surface ocean waves are produced by the wind. As air moves across a smooth ocean surface, capillary waves, characterized by wavelengths less than 1.7 cm and periods less than 0.1 sec, form first. As the wind continues to blow, the capillary waves can grow to become ripples and short choppy waves (Figure 7-6). With increasing wind velocity, duration and fetch (area over which the wind blows), the waves with increasing lengths and heights (and hence energy) are generated contributing to a fully developed sea with white capping (see Figure 7-7).

Figure 7-6. Wind-wave generation process results in a superposition of waves with many different wave lengths and heights in the generation area (WB).

Figure 7.7 The open ocean sea surface is generally chaotic (left and above right) because it is a combination or superposition of many different waves, with different wavelengths and heights. (ltO, LEiO)
The wave energy of a fully developed ocean wave field - with its superposition of waves with many different characteristics - can be partitioned according to its wave period in a wave spectrum (see Figure 7-8).

![Wave Spectrum](image)

**Figure 7-8. Wave Spectrum**
Wave energy, which is proportional to the wave height squared (\( \sim H^2 \)) is distributed in this spectrum according to wave periods, which range from the very smallest period “capillary” waves (T less than < 0.1 sec), to “chop” (0.1 sec < T < 10 sec), to “ocean swell” (10 sec < T < 30 sec), to tsunamis (5 min < T < 1 hr). The forces that cause the waves in specific period ranges are indicated above. Note that most of the wave energy is concentrated in the 1 to 10 second waves. (H0)

In a chaotic ocean, waves interact as they pass through each other. **Constructive interference** (Figure 7-9) occurs when like parts of a wave coincide. The maximum constructive interference is when waves of the same period have their respective crests (or troughs) overlap. The resulting composite wave will have a wave height that is nearly equal to the sum of the wave heights of the two waves. For example, if one wave height is \( H_1 = 3 \) m and the other wave height is \( H_2 = 1 \) m, then the composite wave will have a wave height of almost \( 4 \) m (\( H_1 + H_2 \)) at the moment the two crests and troughs overlap. **Destructive interference** occurs when dissimilar parts of waves coincide. The example in Figure 7-9 assume crest-to-trough alignment, resulting composite wave with a height that equal to the difference in the heights of the respective waves.

Constructive interference of random wave trains explains the unexpected “momentary” and intermittent occurrence of “rogue waves”. However, the highly idealized case study illustrated in Figure 7-9 almost never happens in the real ocean, as waves are usually heading at oblique angles. The example in Figure 7-10 involving waves with different periods and heights, while also idealized, is more realistic. So whenever two or more waves occupy the same space, regardless of the direction in which the waves are traveling, there can be interference. – extreme on the occasion of a rogue wave.
Figure 7-9. (b - left) Constructive Wave Interference. Consider two progressive waves with the same wave periods that are heading in different directions. The waves are aligned crest-to-crest; but have different heights - \( H_1 \) and \( H_2 \), respectively. Where the two waves intersect, there is a “constructive interference” that yields a wave with a wave height = \( H_1 + H_2 \); or the sum of the two.

(c - right) Destructive Wave Interference. Consider the same two progressive waves as in (b). However, the waves are aligned crest-to-trough. Where these two waves intersect, there is a “destructive interference” that yields a wave with a wave height = \( H_1 + (-H_2) \); or the difference between the two.

Figure 7-10. Idealized Rogue Wave. In the real ocean, it is more normal that waves with different periods and different wave heights (e.g., \( H_1 \) and \( H_2 \)) to intersect. So consider two progressive waves - one with a wave period that \( \frac{1}{2} \) of the other’s and heading in different directions. By chance the waves are aligned as shown and where they intersect “constructive interference” results. IN THIS CHANCE CASE, the resultant wave has a momentary wave height about \( 2 \times (H_1 + H_2) \) – perhaps a rogue wave to the unfortunate ship to encounter it.

7-5. PROGRESSIVE WAVE REFLECTION

Wave reflection occurs when a wave rebounds off a steep shore or vertical wall. Wave energy is transformed into a new wave traveling in the opposite direction from which the
original wave came (Figure 7-11). The angle at which the wave strikes (angle of incidence) will be equal to the angle at which the new wave departs (angle of reflection).

![Reflection of progressive waves as viewed from above.](image)

**Figure 7-11.** Reflection of progressive waves as viewed from above.

If the incoming wave strikes head on, then the reflected wave will travel outward in the opposite direction (see Figure 7-12 left). The constructive interference of the two waves produce a standing wave in which the water just moves up and down and back and forth like what one would see in a bathtub (see Figure 7-9 right).

![Reflection of an incoming wave (solid from the right) produces an outgoing wave (dashed to the right). The net result is a standing wave with no net propagation. It is as if there is a second vertical wall at the location indicated by the bold line. (right above) The fundamental “sloshing” mode of a bathtub standing wave or seiche has one node, while (right below) the first harmonic “sloshing” mode of a bathtub seiche has 2 nodes much like our “real ocean” illustration to the left.](image)

**Figure 7-12.** (left) The reflection of an incoming wave (solid from the right) produces an outgoing wave (dashed to the right). The net result is a standing wave with no net propagation. It is as if there is a second vertical wall at the location indicated by the bold line. (right above) The fundamental “sloshing” mode of a bathtub standing wave or seiche has one node, while (right below) the first harmonic “sloshing” mode of a bathtub seiche has 2 nodes much like our “real ocean” illustration to the left.
7-6. PROGRESSIVE WAVE REFRACTION

Shoaling waves that approach a coast are refracted so that they tend to align with the bathymetry and coast. The wave front is bent (leftward toward the beach in the case shown in Figure 7-13) because the inshore end of the wave encounters shallower water depths water and hence slows earlier than the part of the wave front in deeper water.

![Wave Refraction](image)

**Figure 7.13 Wave Refraction**
Waves that approach a beach at an angle (like the above) are refracted so that they align with the local bathymetry wave crest. The blue arrows indicate the energy propagation trajectory (??)

Waves refract so as to concentrate energy on peninsulas and headlands, subjecting them to greater wave-related erosional forces than bays or coves, which are subjected to relatively diminished wave energy. This can be seen by considering a set of equally-spaced orthogonals to the wave crest (Figure 7-14) that define equal amounts of wave energy in deep water and represent the trajectories of the wave energy for different parts of the wave. The wave refraction of the shoaling waves is reflected in the distortion of the orthogonals as they approach the near-shore bathymetry, showing energy concentration (i.e., erosion power) at the headlands.

![Wave Refraction and Energy Concentration](image)

**Figure 7.14 Wave Refraction and Energy Concentration**
As waves approach the shore, they refract – tending to align with the isobaths. This process concentrates wave energy on promontories and diminishes wave energy in bays and coves. (??)
Homework-7: Ocean Waves

Homework-7: Waves EXERCISES

EXERCISE 1 - Wave Generation

Consider wind-generated waves under a variety of circumstances starting with the Southern California scenario in Figure 7-15. Tables 7-1, 2, and 3 provide data for various sets of conditions about which you are questioned.

Figure 7.15 Wind-Generated Waves Scenario
The northerly (i.e., southward-blowing) Santa Ana winds generate waves in the Santa Catalina Channel. However, because it is a fetch-limited generation zone, the waves are likely to be very well sorted according to wave length when they strike the shores of Santa Catalina Island.

Table 7-1
Minimum fetch and duration required for selected wind speeds to set up fully developed seas

<table>
<thead>
<tr>
<th>Wind speed (knots)</th>
<th>Fetch (nautical miles)</th>
<th>Duration (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>20</td>
<td>70</td>
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<td>250</td>
<td>23</td>
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<td>40</td>
<td>710</td>
<td>42</td>
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<tr>
<td>50</td>
<td>1520</td>
<td>69</td>
</tr>
</tbody>
</table>

Table 7-2
Characteristics of waves resulting from selected wind speeds in a fully developed sea

<table>
<thead>
<tr>
<th>Wind speed (knots)</th>
<th>Average height (feet)</th>
<th>Average length (feet)</th>
<th>Average period (seconds)</th>
<th>Highest 10 percent of waves (feet)</th>
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</thead>
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<tr>
<td>10</td>
<td>9</td>
<td>35</td>
<td>2.9</td>
<td>1.8</td>
</tr>
<tr>
<td>20</td>
<td>5.0</td>
<td>113.0</td>
<td>5.7</td>
<td>10.2</td>
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<td>13.6</td>
<td>253.0</td>
<td>8.6</td>
<td>22.6</td>
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<td>40</td>
<td>17.9</td>
<td>446.0</td>
<td>11.4</td>
<td>36.6</td>
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<td>50</td>
<td>48.7</td>
<td>696.0</td>
<td>14.3</td>
<td>49.9</td>
</tr>
</tbody>
</table>

Figure 7.15 Wind-Generated Waves Scenario
The northerly (i.e., southward-blowing) Santa Ana winds generate waves in the Santa Catalina Channel. However, because it is a fetch-limited generation zone, the waves are likely to be very well sorted according to wave length when they strike the shores of Santa Catalina Island.
QUESTIONS

1. The distance from San Pedro, California, to Avalon, on Santa Catalina Island, is about 25 nautical miles (nm). The island is almost due south (180°) from the Los Angeles Harbor at San Pedro (see Figure 7-15).

With reference to Figure 7.15a

a) What is the minimum northerly (from the north) wind speed needed to set-up a fully-developed sea in this channel?

_____________ knots ___________ kilometers/hour.

b) How long must a northerly wind blow in order to generate a fully-developed sea in this channel?

_________________________ hours.
Table 7-3

<table>
<thead>
<tr>
<th>Wind speed (knots)</th>
<th>Fetch (nautical miles)</th>
<th>Duration (hours)</th>
<th>Average height (feet)</th>
<th>Average height (meters)</th>
<th>Average length (feet)</th>
<th>Average length (meters)</th>
<th>Average period (seconds)</th>
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</thead>
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<td>16.8</td>
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<td>8.6</td>
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<td>18.6</td>
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<td>34</td>
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</table>

Figure 7.15b Plot of Table 7-3 Wave Heights Associated with a FDS.
These graphs show the fully-developed sea (FDS) wave height in (above) feet and (below) meters for a given wind speed.
With reference to Figure 7.15b

c) Determine the average wave height that would result in the Catalina Channel from the winds that you determined would set-up a fully-developed sea (answer in part a).

________ feet; ___________ meters;

Would these waves be a problem to boaters in the Catalina Channel?

If not, why not?

With reference to Figure 7.15c

d) Consider the problem of waves encountered by boaters 280 nm off-shore during the winter. What would be the wave height of a fully-developed sea 280 nm south of San Pedro due to the effects of a northerly wind?

________________ meters ___________ feet.

Would this wave height be more or less of a problem for boaters than the wave heights determined in the previous question?
2. In Florida, many of the summer storms that come from the southeast have an enormous fetch. If the average duration of these storms is 2 days, then:

   a) What wind speed is required to set-up a fully developed sea (Table 7-1)?
      ___________________ knots.

   b) What would be the resulting wave height (Table 7-2 or 7-3)?
      ___________________ meters ___________________ feet.
Scientists at Scripps Institution of Oceanography developed Figure 7-16 for the U.S. Army Corps of Engineers, the government agency that is responsible for shoreline protection and harbor improvement for the entire country, including the Great Lakes.

a) The largest waves that strike California during the winter come from what sector?
b) The largest swell that strike California during the summer (surfer nirvana!) come from what sector?

c) What is the range of potential wave periods and heights?

_____________ seconds

_____________ meters

d) Waves with what periods (long, medium, short) produce the highest waves?

4. Storm Waves

a) What is the group speed of storm waves with wave speeds of 100 km/hr?

_____ _____kilometers /hour.

b) How long would it take such waves to reach the New Jersey shoreline from a storm center in the Atlantic Ocean 1500 kilometers away?

_______________ hours.

c) Why are long waves the first to arrive at the coast from an open-ocean distant storm?