CHAPTER 8

ESTUARIES

Estuaries are semi-closed basins in which a rather complex interaction between river inputs, tidal currents and wind leads to the turbulent mixing of salt from the ocean with fresh water and other constituents within the boundaries of the estuary.

To paraphrase Pritchard, a pioneer in studies of estuarine circulation,

"An estuary is a semi-closed coastal body of water in which seawater is measurably diluted by freshwater runoff."

Geomorphologically, estuaries are divided into three categories.

> Coastal plain estuaries are drowned river valleys whose depths are shallow and whose bottom is gently sloping.
>
> Bar-built estuaries are almost totally enclosed by outer sand bars; e.g. along the New Jersey and North Carolina coasts.
>
> Fjords are a product of glacial erosion with steep sides and large depths except at the sill near the entrance.

Circulation and mixing within a particular estuary is controlled by the tidal prism volume, P, to river flow volume, R, ratio. The tidal prism is defined as the total volume of water that enters and leaves the estuary each tidal cycle, while R is the volume of river input during the same period.

Salt Wedge Estuary

When $P/R << 1$, density differences are important in driving the circulation. If there were no friction in the 2-layer, then the river water would flow out of the estuary without mixing with the water as shown in Figure 8.1. The cross-estuary geostrophic balance would lead to the interface configuration shown in the cross-section.

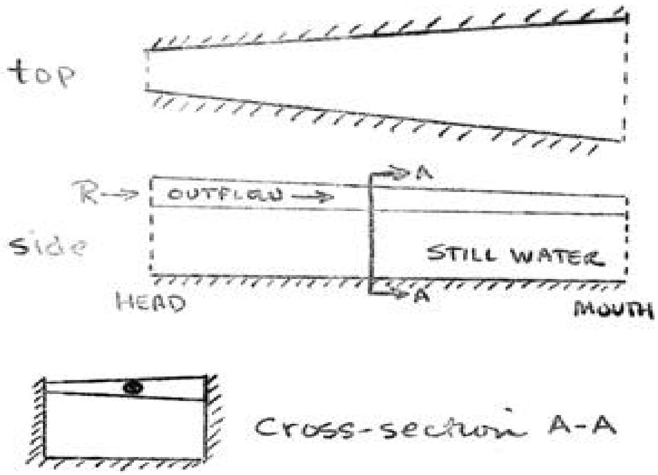

Figure 8.1 Schematics of sea level distribution associated with a non-tidal estuary that is dominated with freshwater inflow and no friction.

With <u>friction</u> considered, the more realistic <u>salt wedge</u> arises because a pressure gradient is required to balance the frictional resistance that accompanies turbulent entrainment and mixing at the freshwater/saltwater interface. The pressure gradient is maintained by the saltwater in flow.

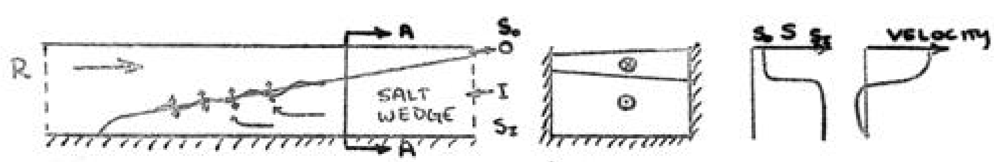

Figure 8.2 Salt wedge estuary with friction

Such a system looks like the figure above which shows the river inflow R, the salt (S_I), incoming ocean water at rate I, mixing through the salt wedge interface with the out flowing freshwater. The mixture, with salinity S_o, eventually exits the mouth at a rate 0.

A box model can be developed for this system in order to relate the transports R, O and I and the salinities S_o and S_I. In simplest terms it looks like that shown in Figure 8.3.

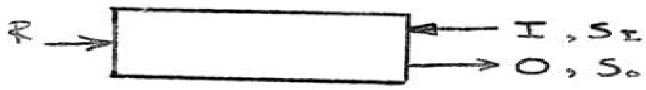

Figure 8.3. Box model of salt wedge estuary.

Mass balance requires

$$O = I + R$$

and salt balance requires that

$$S_o O = S_I I$$

since no salt is added at the head. These relations can be combined as

$$O = \left(\frac{S_I}{S_I - S_o} \right) R \, ,$$

which shows that, as $S_I \rightarrow S_o$, the estuarine outflow O becomes much greater than the river inflow R. This occurs because large amounts of saltwater are being entrained and mixed with the outgoing river flow.

Partial Mixed Estuary

If P/R is order 1 (or $0.5 < P/R < 5$), then tidal flow effects become important and we have what is called a partially-mixed estuary. For this case and the next case, where tidal effects become dominant, the important dynamic balance is between a sea level-slope induced pressure gradient and bottom friction. Associated with the bottom friction are the turbulent eddies which are effective in mixing salt upwards and increasing the potential energy. If the tidal motions are averaged the remaining mean distributions of salt and velocities for a typical narrow estuary are shown in figure 8.4.

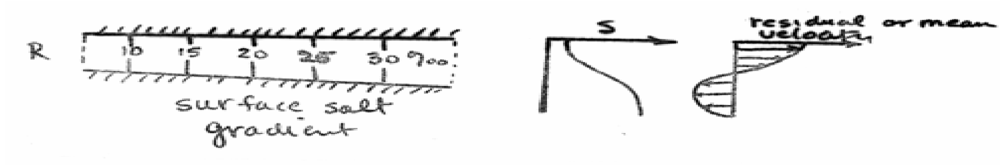

Figure 8.4 Salt and velocity distribution in a narrow partially mixed estuary

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For a wide estuary Coriolis forces causes a non-uniform distribution of tidal flow which can be decomposed as shown in Figure 8.5.

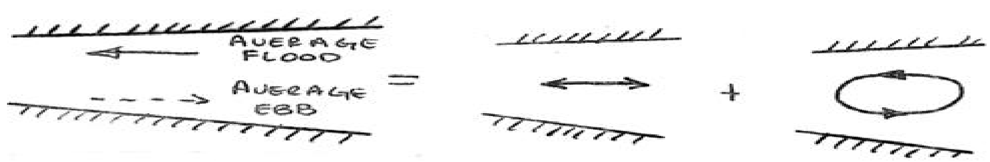


Figure 8.5 Tidal circulation decomposition in a wide estuary.

The circulation imposed by rotational effects leads to a lateral asymmetry in the surface (and at depth) salt distribution as shown in Figure 8.6.

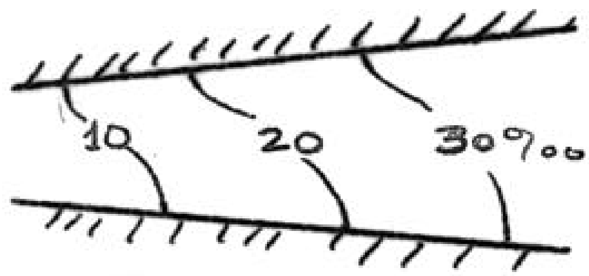


Figure 8.6 Salt distribution in partially-mixed wide estuary

Well-Mixed Estuary

When $P/R >> 1$, tidal flow effects dominate the circulation and mixing characteristics of the estuary. Further, if (1) the tidal prism volume is of the same order as the mean volume of the estuary and (2) the estuary is narrow, then a well-mixed estuary like the Piscataqua River and Little Bay sections of the Great Bay, New Hampshire results. In such cases, the basin geometry strongly influences on the details of the tidal flow, which results from the combined effects of the tidal range, imposed at the mouth of the estuary, and the typical cross-sectional area of a particular estuary. In this case vertical salt and velocity distributions are nearly uniform and the principal horizontal salt gradients are confined to regions near the river mouths as shown in Figure 8.7.

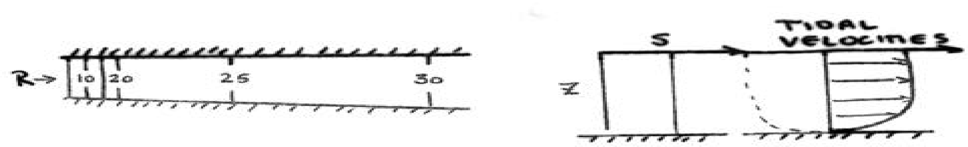


Figure 8.7 Salt and velocity distribution in a well-mixed estuary.

The upstream salt flux in this case is caused by turbulent diffusion and is balanced by the downstream advection of fresh river water.

A more formal classification scheme for estuaries has been proposed by Hansen and

Rattray (1966)[1], who have defined a set of parameters that determine the amount of energy dissipation in the tidal wave. The higher the dissipation the greater the progressive wave character exhibited by the tidal wave. Most estuarine tides exhibit a combined progressive/standing wave character with a sea level-velocity phase difference falling somewhere between 0 and $p/2$ radians.

The Hansen and Rattray classifications are based on the stratification and circulation parameters discussed next.

<u>Stratification Parameter</u> - The *densimetric Froude number* is defined as

$$F_m = \frac{U_f}{U_d},$$

where U_f is the characteristic or "typical" river flow velocity, defined as

$$U_f \equiv \frac{R}{A}$$

for a river discharge rate R and a cross-sectional area A;

where U_d is the densimetric velocity defined as

$$U_d \equiv \frac{R}{\sqrt{\frac{\Delta r}{r} gD}}$$

for a seawater/river water density difference, Δr, and a characteristic depth, D, with a gravitational acceleration g.

<u>Circulation Parameter</u> - P is defined as

$$P = \frac{U_f}{U_t},$$

where U_t is the root mean square (rms) tidal current speed, according to

1.Limnology and Oceanography, 11(3).

$$U_t = \left[\frac{1}{T_{M_2}} \int_0^{T_{M_2}} \left(a \cos \frac{2\pi}{T_{M_2}} t \right)^2 dt \right]^{1/2} = \frac{a}{\sqrt{2}} \quad .$$

Hansen and Rattray have related the two parameters F_m and P theoretically to a salinity difference parameter $\delta S/S_o$ (where dS is the top to bottom S difference and S_o is the cross-section mean S) and the current ratio U_s/U_f (where U_s is a tide averaged surface current). The results of these computations are summarized in Figure 8.8, which is also subdivided into their estuarine classifications.

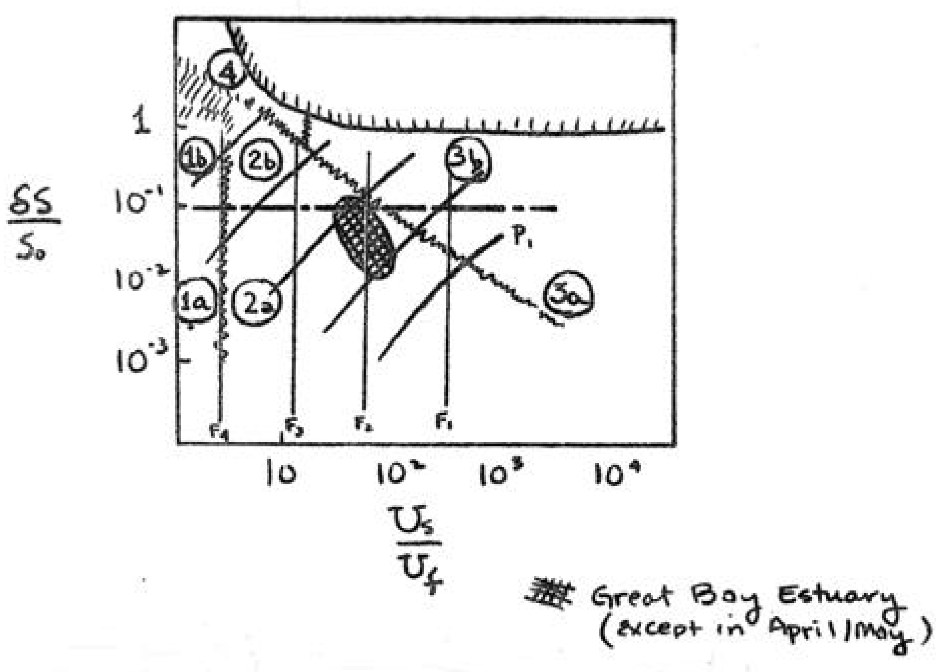


Figure 8.8 Hansen-Rattray estuarine classification diagram.

Classifications are based on both the vertical distribution of tidal averaged or residual velocity and the degree of vertical stratification with (a) referring to slight stratification and (b) appreciable stratification.

A <u>Type 1</u> estuary (according to this scheme) is characterized by a seawater net flow at all depths. Thus an upstream salt flux is achieved through a turbulent diffusion process.

A Type 2 estuary exhibits a reversal of net flow at some mid-depth. Therefore the upstream salt flux is achieved through a combination of advection and diffusion.

In a Type 3 estuary the upstream salt flux is primarily advective. Fjords usually fall into category 3b for which the lower layer is so deep that the density driven circulation does not penetrate to the bottom.

A Type 4 estuary is a salt wedge estuary with circulation patterns (discussed previously) in which the principal mixing between salt and fresh water occurs along the fresh/salt water interface.

In practice it is not unusual for different parts of the same estuary to be classified differently. A case in point, the different parts of the Great Bay Estuary plot in the cross-hatched area on the Hansen-Rattray stratification-circulation diagram. Even these Great Bay classifications migrate from the 2a into the 2b (increased stratification) area during the spring runoff period. These data, computed by Arellano (1977), are presented in Table 8.1 for the estuarine locations shown in Figure 8.9.

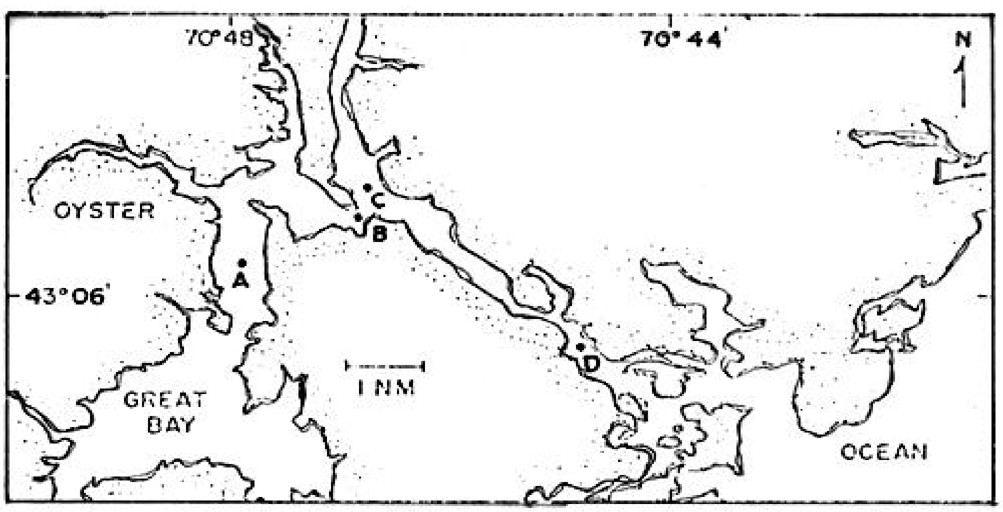


Figure 8.9 Locations where the estuary is classified.

Table 8.1 Values of Parameters Inferred from Distribution of Velocity and Geomorphology in Particular Locations in the Estuary

Locations	A	B	C	D

Low River Flow Parameters				
U_d	1.13	1.41	1.55	1.97
P	1.41×10^{-3}	8.80×10^{-4}	3.40×10^{-3}	1.51×10^{-3}
F_m	4.6×10^{-4}	6.7×10^{-4}	1.05×10^{-3}	5.48×10^{-4}
U_s/U_f	100	90	45	90
d_s/S_o	0.030	0.014	0.035	0.022
High River Flow Parameters				
P	7×10^{-3}	4.45×10^{-3}	1.7×10^{-2}	7.5×10^{-3}
F_m	2.3×10^{-3}	3.35×10^{-3}	7.5×10^{-3}	2.74×10^{-3}
U_s/U_f	30	28	18	31
d_s/S_o	0.11	0.035	0.14	0.12

Now consider the important dynamical considerations which control both the tidal flow and the residual flow. In a well-mixed estuary, such as the Great Bay with $P/R >> 1$ and $P/V = 0(1)$, tidal pressure gradients associated with sea level tilts, $r\, g\, \partial h / \partial x$, are balanced by bottom stress, t^b, as derived below.

By assuming $\partial u / \partial t$ is negligible, the longitudinal momentum equation is

$$0 = -\frac{1}{r}\frac{\partial p}{\partial x} + \frac{t_x^s - t_x^b}{h},$$

where t_x^s is the surface stress and h is the water depth.

Assuming a hydrostatic pressure field and integrating vertically we find that

$$0 = -r\, g \frac{\partial h}{\partial x} + \frac{t_x^s - t_x^b}{h}.$$

For $t_x^s = 0$ and $t_x^b = t_b$

$$\frac{t_b}{h} = r\,g\,\frac{\partial h}{\partial x} \quad .$$

For the Great Bay Estuary $\partial h / \partial x$ fluctuates in the range $\pm 1.5 \times 10^{-5}$ with the frictional effects are most evident in the velocity profile 1m above the bottom as shown in Figure 8.10. (Refer to the discussion regarding frictional effects and velocity profiles.)

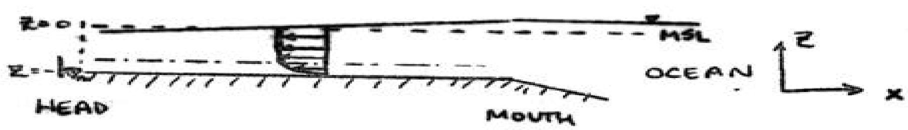


Figure 8.10 Schematic of longitudinal pressure gradient driven flow.

The residual flow in a well-mixed estuary is influenced by density-induced pressure gradients and controlled by bottom friction. The longitudinal cross-section of salinity (or density) of such an estuary looks something like the sketch in Figure 8.11.

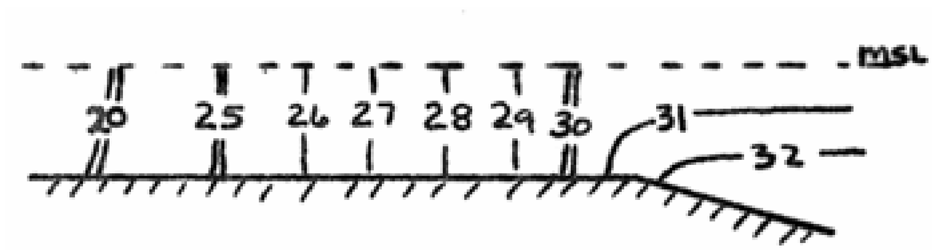


Figure 8.11 A longitudinal section of salinity profiles in a well-mixed estuary.

A salinity distribution like this leads to horizontal density gradients $+\partial r / \partial x$, which give rise to horizontal pressure gradient forces $gz(\partial r)/\partial x)$ which vary with depth. Continuity requires that some mid-depth <u>isobar</u> be level. Thus the pressure distribution can be drawn as seen in Figure 8.12.

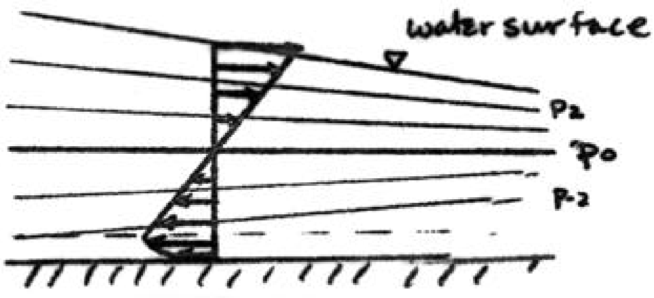


Figure 8.12. Pressure and associated velocity distributions, due to the seaward salinity gradient in a well-mixed estuary (see Figure 8.11).

The corresponding density-driven (or residual) velocity distribution is indicated.

Of course friction controls the overall magnitude of the residual circulation and its effects are seen in the velocity profile near the bottom.

The total estuarine circulation is the superposition of <u>fluctuating</u> tidal flow and a more <u>steady</u> density-driven residual flow. The relative magnitude of the different components depends upon considerations discussed previously. In each case the dynamics are influenced by bottom friction.

In the course of our initial discussions of bottom friction we showed how the depth dependence of the mean velocity in the bottom boundary layer could be expressed in terms of the bottom stress, $t_b = r\, u_*^2$, the bottom roughness, z_o, and the Von Karmen constant $k_o = 0.4$ according to

$$\overline{U}(z) = \frac{u_*}{k_o} \ln\left(\frac{z + z_o}{z_o}\right),$$

This is the mathematical expression for the mean velocity in a “log layer”. Since it is difficult to measure the bottom stress directly, this relation can be manipulated to provide an indirect means of determining stress. Rearranging the relation above yields

$$\ln(z+\ z_o) = \frac{k_o}{u_*}\overline{U} + \ln\ z_o\ ,$$

where now the coefficients $\frac{k_o}{u_*}$ and ln z_o can be determined from measurements of the mean velocity profile $\overline{U}(z)$. As a first approximation assume $z >> z_o$ on the left hand side of the relation above and plot values of $\overline{U}$ versus ly z. A linear regression of these results will yield a first approximation to the coefficients

$$\left(\frac{k_o}{u_*}\right)_1 = \frac{k_o}{u_*}\ \text{and}\ \ (z_o)_1 = z_o\ .$$

A second interaction of the same procedure plotting $\ln(z+(z_o)_1)$ versus $\overline{U}(z)$ will produce better estimate $(z_o)_2$ and $\left(\frac{k_o}{u_*}\right)_2$ from which $(t_b)_2$ can be computed. This procedure converges rapidly usually so that two interactions are all that are usually necessary. Thus, using this reasonably simple technique, estimates of the frictional character of a flow regime can be made. The physical interpretation of the z_o estimate is more difficult as it includes the effects of topographic roughness as well as hydrodynamic turbulence and sediment load.

Mixing and Transport of Biogeochemical (BGC) Constituents

We have already shown that, in a turbulent environment, momentum exchange is best described in terms of eddy exchange between layers as measured by eddy coefficients of viscosity,

$$A_z = \frac{-\,r\,\overline{u'w'}}{\partial\overline{U}/\partial z}\ .$$

Therefore we would expect that the transfer of any BGC constituents could be described in similar terms. Let's explore that possibility by assuming a concentration distribution of dissolved or suspended constituent c = c(x,y,z,t), which can be decomposed into mean and fluctuating components, according to $c = C + c'$. The vertical transport of c (or Γ_c) is related to the mean vertical concentration gradient according to

$$\Gamma_c = -K_{cz}\frac{\partial C}{\partial z},$$

where K_{cz} - the eddy diffusion coefficient for the vertical transfer of the constituent - is

$$K_{cz} = \frac{\overline{w' c'}}{\partial \bar{c} / \partial z} \quad .$$

In the presence of density stratification, we expect that water parcels, which are displaced vertically from their mean position, will exchange momentum more rapidly than constituent transfer. Thus $|K_z| < |n^e{}_z|$ (different units) and constituent transfer is incomplete before water parcels begin to return to their mean level. Because horizontal transport is less inhibited, so

$$|K_{cx}| \; a \; |n^e{}_x| \qquad \text{and} \qquad |K_{cy}| \; a \; |n^e{}_y|$$

Without stratification, we expect that

$$|K_z| \, a \, |n^e{}_z| \quad .$$

We will now incorporate these ideas into the development of the conservation of a BGC constituent in a turbulent hydrodynamic environment.

The general Eulerian conservation statement for a general constituent

$$\underset{(a)}{\frac{Dc}{Dt}} = \underset{(b)}{D\nabla^2 c} + \underset{(c)}{f_c}$$

where term (a) is the *substantial rate change* of c or

$$\frac{Dc}{Dt} = \frac{\partial c}{\partial t} + (\vec{V} \bullet \nabla)\, c\,;$$

and term (b) is the molecular diffusion; and term (c) is a source (or sink) term.

Decomposing the velocity and constituent field into their mean and turbulent components according to

$$\vec{V} = \vec{V} + \vec{v}'\,;\; c = C + c'$$

and assuming that turbulent transport is more important than molecular transport

(and thus $K_c >> D$), the conservation statement can be written

$$\frac{\partial C}{\partial t} = -[\frac{\partial UC}{\partial x} + \frac{\partial VC}{\partial y} + \frac{\partial WC}{\partial z}] \qquad \text{advection of C by the “mean flow”}$$

$$+ \frac{\partial}{\partial x}(K_{cx}\frac{\partial C}{\partial x}) + \frac{\partial}{\partial y}(K_{cy}\frac{\partial C}{\partial y}) + \frac{\partial}{\partial z}(K_{cz}\frac{\partial C}{\partial z}) \qquad \text{eddy diffusion}$$

$$+ f_c \qquad \text{source/sink}$$

Now consider the application of the above

relation to a well-mixed estuary, like the Great Bay NH, where turbulent exchange processes are more important than advective exchange over the time scale of days. Then the above unsteady eddy diffusion of a constituent in the above relation can be simplified to

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}(K_{cx}\frac{\partial C}{\partial x}) + \frac{\partial}{\partial y}(K_{cy}\frac{\partial C}{\partial y}) + f_c$$

or in one-dimension, without sources, it would be

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}(K_{cx}\frac{\partial C}{\partial x}).$$

In the latter case, the accumulation of C at a particular location in the estuary is determined by the difference between the eddy transport into and out of the region... a reassuring result.

Consider the application of the 2-D relationship to the situation, where $f_c = 0$; and at t = 0^+ a finite amount - C_o ($[C_o]$ = mass/unit depth) - of a constituent is injected at the location x = 0 and y = 0. The relevant conservation statement is

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}(K_{cx}\frac{\partial C}{\partial x}) + \frac{\partial}{\partial y}(K_{cy}\frac{\partial C}{\partial y})$$

Its solution, under the constraint that the concentration must remain finite at large distances x and y, yields Fick's Law, i.e.

$$c(x, y, t) = \frac{C_o}{4\pi\, t(K_{cx}K_{cy})^{1/2}} \exp(-\frac{x^2}{4K_{cx}t} - \frac{y^2}{4K_{cy}t})$$

You should recognize the so-called Fickian solution (see Figure 8.13)

as a time dependent Gaussian distribution.

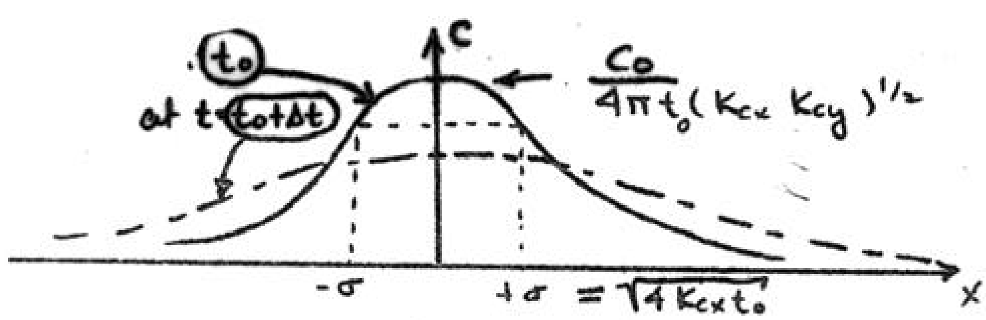


Figure 8.13. The x-distribution of the Fickian concentration c (x) a constituent at the initial time (solid) and some time later (dash-dot).

Applying these ideas to a *model estuary*, enables us to compute the distribution of a constituent as a function of flow and mixing parameters. In this case our *model estuary* is defined by an input flow of V_i and concentration C_i and an output flow of V_o and C_o.

The distribution of C can be determined rather easily for a one-dimensional model estuary, if it is assumed that (1) advection is unimportant relative to diffusion and (2) the diffusion coefficient K and the density ρ are constant throughout the estuary all the time (see Figure 8.14).

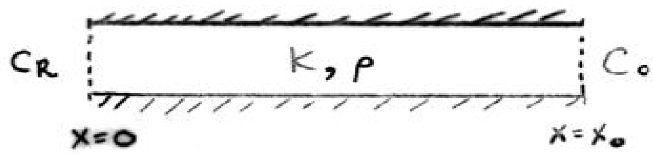


Figure 8.14 A one dimensional estuary with spatially constant diffusion K and water density ρ. The concentrations at the two ends C_R and C_o are held constant.

The appropriate conservation equation, in this case, is

$$\frac{\partial C}{\partial t} = K\frac{\partial^2 C}{\partial x^2} + \frac{\tilde{P}}{\rho} \qquad ,$$

where $\tilde{P}$ is the mass gain or loss rate per unit volume.

The boundary conditions are that

$$\text{at } x = 0;\ C = C_R \qquad \text{and} \qquad \text{at } x = X_o;\ C = C_o$$

For a steady state case

$$\frac{\partial^2 C}{\partial x^2} = -\frac{\tilde{P}}{\rho K}.$$

If it is assumed that $\tilde{P}$ is constant throughout the estuary (as it might be for sediment resuspension or deposition), then the above relation is easily integrated twice, resulting in

$$C = \frac{-\tilde{P}}{2\rho K}x^2 + ax + b.$$

where a and b are determined from boundary conditions. Applying the boundary condition at x = 0 ($C = C_R$) implies that $b = C_{R.}$ Applying the boundary condition at x = X_o ($C = C_o$) gives

$$C_o = -\frac{\tilde{P}}{2 r K} X_o^2 + a\, X_o + C_R$$

or solving for a gives

$$a = \frac{C_o - C_R}{X_o} + \frac{\tilde{P}\, X_o}{2 r K}.$$

so that the solution is

Therefore

$$C(x) = C_R + \left(\frac{C_o - C_R}{X_o} + \frac{\tilde{P}\, X_o}{2 r K}\right) x - \frac{\tilde{P}}{2 r K} x^2 .$$

For a conservative constituent there is no source or sink, i.e. $\tilde{P} \equiv 0$, the concentration distribution is *linear in x*, according to

$$C(x) = C_R + \left(\frac{C_o - C_R}{X_o}\right) x \; .$$

If we consider the distribution of salinity S, for which $C_R = 0$ and $C_o = S_{o,}$ then the relation becomes

$$S(x) = \frac{S_o}{X_o} x$$

or that the longitudinal salinity gradient in the estuary is

$$\frac{dS}{dx} = \frac{S_o}{X_o} \equiv \text{a constant} \; .$$

The above result can be used in evaluating the applicability of this model to an estuary. For example, if the geographical distribution of salt is nearly linear, then assuming K is approximately constant is valid and the theory can be applied to the real estuary in question.

Further, if the flow rate of freshwater into the estuary is known, then the value of K can be computed. This can be done by expressing the constant salinity gradient in terms of the freshwater concentration gradient, by first defining the freshwater fraction

$$F = \left(\frac{S_o - S}{S_o} \right).$$

At the head of an estuary the river inflow has a near-zero salinity, i.e. S = 0; so that F = 1. At the mouth the water has a salinity S_o, thus F = 0. The gradient dF/dx is

$$\frac{dF}{dx} = -\frac{1}{S_o} \left. \frac{dS}{dx} \right|_{estimated}$$

At the mouth of the estuary we know that the concentration of freshwater $C_R^F = 1$ and

$$R\, C_R^F = -K \frac{\partial F}{\partial x}.$$

Therefore

$$K_{est.} = -\frac{R}{\frac{dF}{dx}} = \frac{RS_o}{\left. \frac{dS}{dx} \right|_{estimated}}$$

.

where $[K] = \frac{L^3 T^{-1}}{L^{-1}} = L^4 T^{-1}$.

CHAPTER 8 PROBLEMS

Problem 8.1. Estuarine Dynamics

If the

- mean river flow into the inner harbor is 5 m^3/sec;
- volume of Boston Inner Harbor is approximately $1.5x10^8$ m^3;
- mean salinity is 30.5 psu; and
- salinity of the 'ocean' adjacent to the Harbor is 31 psu,

then what is the *mean residence time* for a parcel of river water that enters at the head of the estuary?

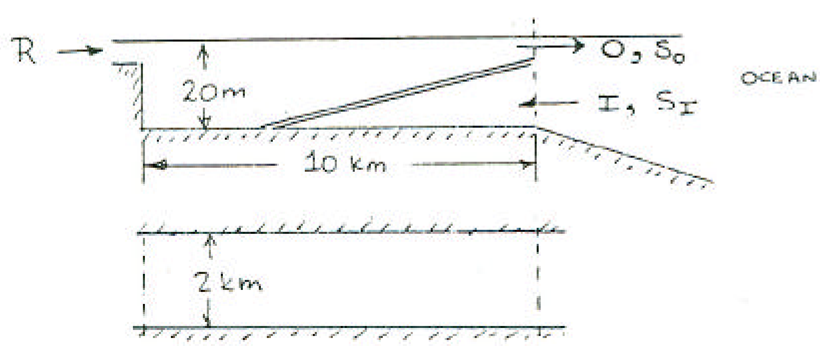
Problem 8.2. Estuarine Salt Transport

In your own words, describe four types of estuaries, including some explanation of how salt (and other materials in the water) are transported.

Problem 8.3 Estuarine Mixing

a) Discuss and contrast the density driven (baroclinic) flow in highly stratified and vertically well-mixed estuaries.

b) Consider the simple salt wedge estuary shown below, with transports for the river = R, deep inflow = I, and surface outflow = O, each with corresponding salinities S_I and S_0 respectively.



c) Using ideas of conservation of salt and volume, derive the general relation between inflow transport I and river flow transport R and the inflow and outflow salinities, S_I and S_0.

Given S_0 = 32.0 psu, S_I = 33.0 psu, and R = 3 x 10^3 m^3/sec, what is I ?

Problem 8.4 Estuarine Friction

It is sometimes useful to parameterize the friction force F_f as:

$$F_f = \frac{\tau_O}{h} = C_D \rho \frac{U_b^2}{h} \quad ,$$

where C_D is a drag coefficient, U_b is the depth-averaged velocity, ρ is the mean water density, h is the depth of the estuary;

With the purpose of studying estuarine bottom friction, you obtain the mean current profile measurements below at Site 1, where water depth h = 15m, and Site 2 where h = 5m.

Height Above Bottom (cm)	**Mean Velocity (cm/sec) Site 1**	**Site 2**
5	35.1	11.2
15	56.0	17.3
25	66.2	20.2
60	84.5	25.6
100	93.3	28.7
200	110.0	32.8

a) Determine the roughness length z_o, the friction velocity, u_*, and the bottom stress, τ_O, at the two sites.

b) Determine the drag coefficient C_D from both sets of measurements assuming the flow is constant at depth above 2 meters. What does the difference between your results suggest about this form of the parameterization?

c) Assuming that the pressure gradient force and the bottom friction force are balanced, use your estimate of the depth averaged velocity and the drag coefficient to calculate the local slope of the water surface at the two sites. What is the elevation change per kilometer downstream?

d) Calculate the local transverse surface slope of sea level at each of the sites assuming that a geostrophic balance exists. What is the ratio of the transverse to longitudinal (downstream) surface slopes (Use answer to part c). What is the difference in elevation per kilometer in cross-stream direction at the two sites?

Problem 8.5 Estuarine Dispersion

Considering dispersion only in the bottom boundary layer (see below), it has been found in many cases that horizontal turbulent diffusion coefficients depend on the friction velocity u_* (i.e. bottom stress) and water depth h. Laboratory experiments in channels , for which w >>h, indicate that the horizontal turbulent diffusion coefficients are anisotropic and can be parameterized as

$$K_x = 5.9\, u_* h$$
$$K_y = 0.23\, u_* h$$

Apply these results to the estuarine channel shown below, where w = 500 m and depth h = 5 m. Given that an amount of dye A_o is placed in the boundary layer at x = y = 0 at t = 0, use u_* = 4 cm/sec and plot the $A_o \times 10^{-9}$ contour for t = 100 seconds after dye injection. What is the concentration at x = y = 0 after 100 seconds?