CHAPTER 1 - GEOPHYSICAL PERSPECTIVE

GEOGRAPHICAL ISSUES

From the point of view of the Earth as a whole, the world's oceans (Figure 1.1) occupy relatively shallow indentations on the earth's surface. In fact if the Earth were shrunk to the size of a basketball the presence of the oceans would make the basketball feel damp! In what follows we explore (a) the "thinness" of the oceanic layer relative to other layers which form the Earth, (b) the reasons for the existence of ocean basins and finally (c) the nature of the shape of the Earth.

As shown in Figure 1.2, there are several distinct layers to the inner earth structure as inferred by geophysicists (see Table 1.1).

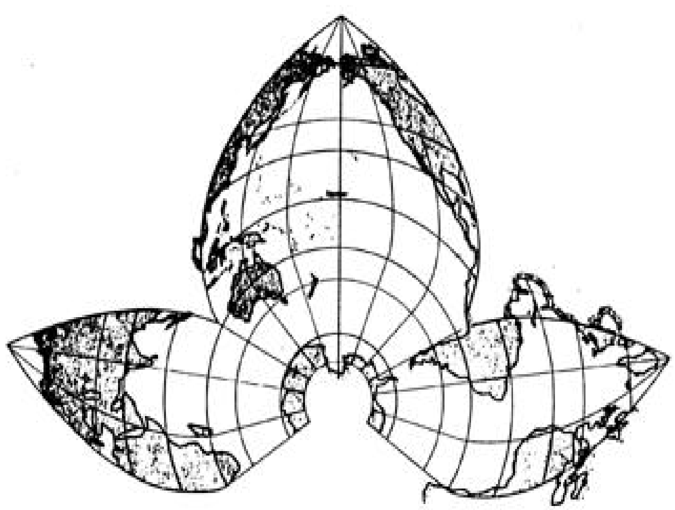

Figure 1.1. The world ocean in Bartholomew's petal projection. [Adapted from Plate 2 of *The Times Atlas of the World*, Vol. 1.] (Von Arx, 1974)

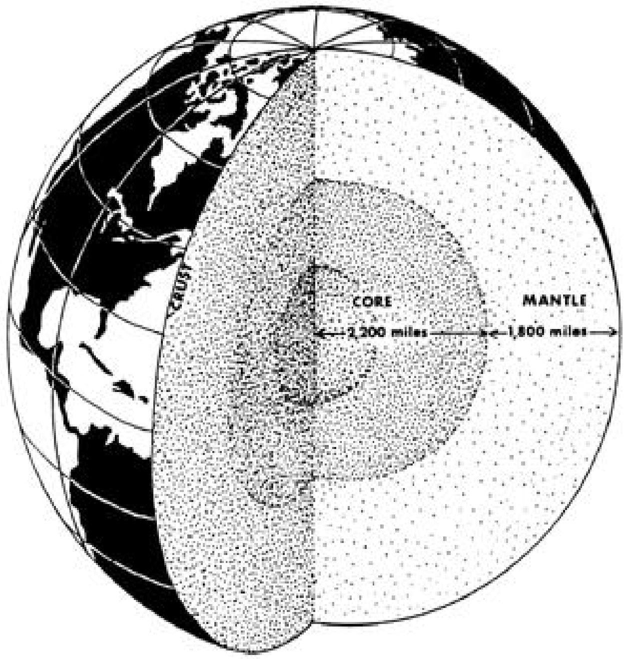

Figure 1.2. Layers of the earth. The core, about 7,000 km. (4,400 miles) in diameter, consists of exceedingly hot, dense material. It is thought to have two parts: a solid center and a liquid outer layer. Surrounding the core is the mantle, about 2,000 km. (1,800 miles) of dense rock. Next is the crust, or outermost layer of the earth. This is very thin in comparison, its thickness varying from as little as 5 km. (3 miles) to at most 50 km. (30 miles). (Ericson & Wollin, 1967).

The most striking feature of Table 1.1 is the *increase of density with depth.* This understanding along with other geophysical data including the relatively *small gravity anomalies* observed over the Earth's surface has led scientists to conclude that these layers of the Earth are in *isostatic equilibrium*. The situation of *isostasy* is consistent with the fact that on average layers within the Earth "float on one another". (The special case of this situation in water is *hydrostatic equilibrium*).

Isostasy and the large difference in thickness between continental and oceanic crust explains the existence of ocean "basins". Archimedes principle tells us two things; (1) a piece of buoyant (i.e. floating) material, such as wood in water, that is less dense than an equally thick piece of more dense material will rise higher in a fluid and (2) a piece of buoyant material that is thicker than a thinner piece of equally dense material will also rise higher in a fluid. As shown in Figure 1.3, the thicker continental crust "floats" higher in the mantle below relative to the thinner oceanic crust. The density differences of the earth crustal layers are so small that they are much less important than layer thickness differences. (The Theory of Seafloor Spreading and Plate Tectonics explains why the densities and thicknesses of the different crustal layers differ).

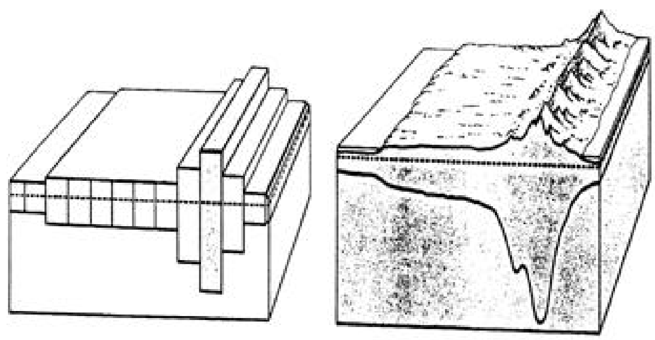

Figure 1.3 (right) A hypothetical section of the earth's crustal layer – as defined at depth by the "Moho" discontinuity. Note that the continental crust in the mountain region plunges much further into the mantle than does the relativey thin oceanic crust to the left. (left) This earth geological configuration can be modeled with slabs of wood floating in a fluid.

To understand the physics of this and other Earth configurations, we need to know a little bit about how the Earth responds to different forces. When sustained loads are applied over *geologic time scales*, Earth material flows like a fluid and undergoes permanent deformation sometimes known as *"plastic deformation."* The same kind of behavior is observed when loads, which have been applied for long periods of geologic time, are "suddenly" released. For example, Scandinavia is still rising at the rate of about 1 cm per year in response to the melting of the last glacier 10,000 years ago.

When loading is applied to the Earth on time scales that are short compared to geological time scales, the Earth exhibits *"elastic deformation"* – that is it distorts and springs back to its original configuration. For example, the Earth's surface temporarily bulges outward about 1 meter in response to moon- and sun-induced tidal forcing.

How are these ideas relevant to the Earth's surface configuration in general and the oceans in particular? First we know that the earth is almost a sphere. In fact, the 3rd century B.C. Greek astronomer Eratosthenes assumed a spherical Earth and estimated its circumference reasonably accurately. (I wonder if the sailors between his and Columbus time knew of this computation).

The Earth, however, is actually an ellipsoid with a polar radius which is 22 km less than the equatorial radius. To understand why, consider a rotating spherical earth (Figure 1.4). Here each "parcel" of earth material is acted upon by centrifugal force (per unit mass) of $\vec{c}$, a vector that can be resolved into its locally vertical $\vec{c}_v$ and horizontal $\vec{c}_h$ components (see the Appendix). The vertical component $\vec{c}_v$ can not move an earth parcel upward (hence no distortion) because it is opposed by the very large gravitational force per unit mass (i.e. $|\frac{\vec{c}_v}{\vec{g}}| \prec\prec 1.$). On the other hand, $\vec{c}_h$ is unopposed - a non-equilibrium situation in which Earth material will be forced to move horizontally equatorward until a non-spherical, dynamic equilibrium Earth configuration arises.

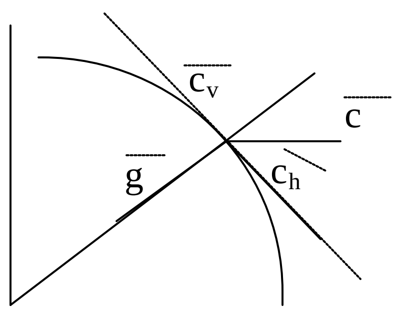

Figure 1.4. Centrifugal force due to earth rotation results in an unbalanced horizontal force component c_h on a spherical earth.

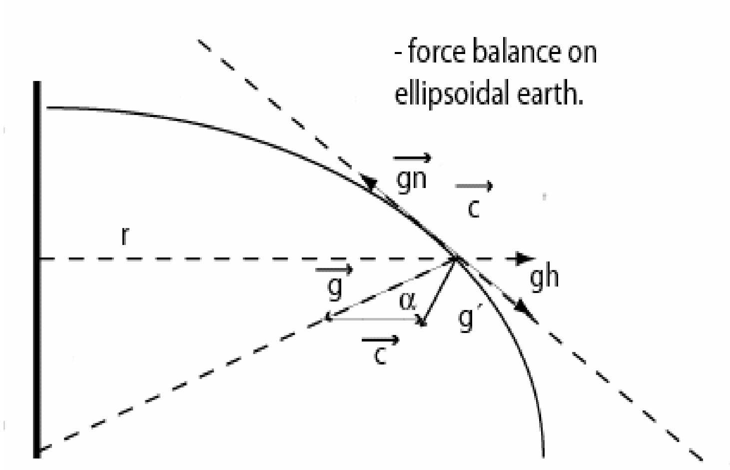

Figure 1.5. Force balances for an rotating ellipsoidal Earth in dynamic equilibrium. Note that the local tangential components of the geocentric gravitational force (per unit mass) g_h and centrifugal force c_h are equal.

As a consequence of this tendency for deformation toward a dynamic equilibrium configuration, the Earth has had an ellipsoidal shape throughout its approximate 13 billion year history (see Figure 1.5). The actual dimensions of the ellipsoidal shape have changed with time because the Earth rotation rate has been slowly decreasing since its formation (due to ocean tidal friction).

Why an ellipsoidal shape? It turns out that the dynamic equilibrium configuration of a rotating Earth is one with an elliptical cross-section. The surface Earth material does not move in this equilibrium configuration. This occurs because the component of geocentric gravitational force (*per unit mass*) that is tangent to the Earth's surface $\vec{g}_h$ is <u>equal and opposite</u> to the tangential component of the "centrifugal force" (*per unit mass*) $\vec{c}_h$. Note that the "effective gravitational force" $\vec{g}'$ (i.e. locally perpendicular to the Earth surface) is the <u>vector sum</u> of the geocentric gravitational force, $\vec{g}$, and the centrifugal force $\vec{c}$. Because the centrifugal force is so much smaller than the gravitational force, the geometrical angle a between , $\vec{g}$, and $\vec{g}'$ is very small. (Later you will have a chance to show that the angle is less than a degree). Due to an increased centrifugal force at locations closer to the equator and the larger distances from the Earth's surface to the center (see Figure 1.6) the effective gravitational acceleration $\vec{g}'$ decreases by 0.3% (or 3 cm/sec^2) going from pole to equator.

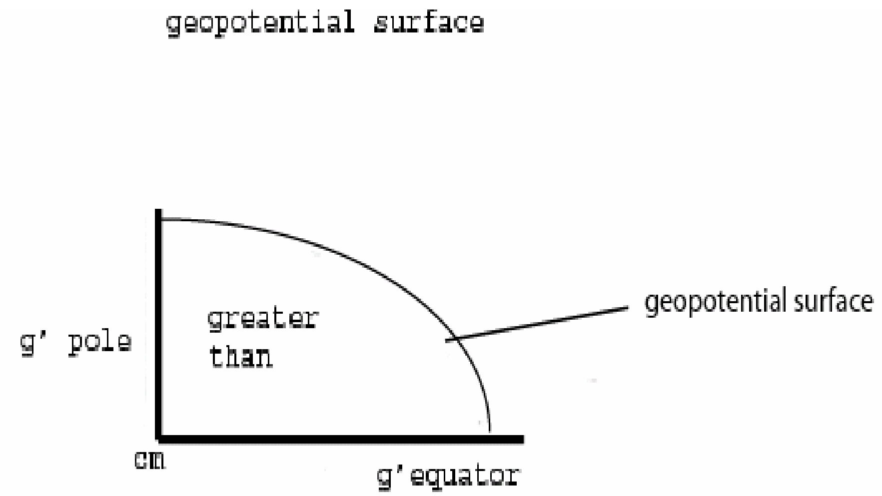

Figure 1.6. The variation of the effective gravitational force per unit mass on the ellipsoidal Earth.

Under these circumstances, the Earth including the ocean surface is a surface of constant geopotential (or a surface that has a uniform gravity potential above the Earth's geocenter). It turns out that the true equilibrium shape of the Earth (called the *geoid*) departs from the ideal ellipsoid because Earth material is not distributed homogeneously. For example, the sea surface departs from the ideal ellipsoid by nearly 30m in the Indian Ocean. It is this "bumpy" geoid that is composed of motionless parcels that is the reference surface of the ocean.

That ocean water parcels making up the geoid will remain motionless is a consequence of there being part of a *geopotential surface* (that is a surface that has a uniform gravity potential above the Earth's geocenter). This means that the geoid is composed of Earth parcels for which it takes equal amounts of work to move them from the center of mass of the Earth to the surface against effective gravity g' according to

$$Work = \vec{F} \bullet \vec{r} \quad ,$$

where F = g' and r is the radial distance from the Earth's center of mass.

HEAT BUDGET OF THE EARTH SYSTEM

The sun is the ultimate source of energy* for the Earth system. Overall there is a balance between incoming solar short wave energy intercepted by the Earth and the outgoing long wave energy being reradiated by it. (Figure 1.7). We know this because the annual mean temperature of the Earth System as a whole is nearly constant. This does not mean that this balance is achieved everywhere. and in fact the radiant energy reaching the Earth's surface is quite nonuniform.

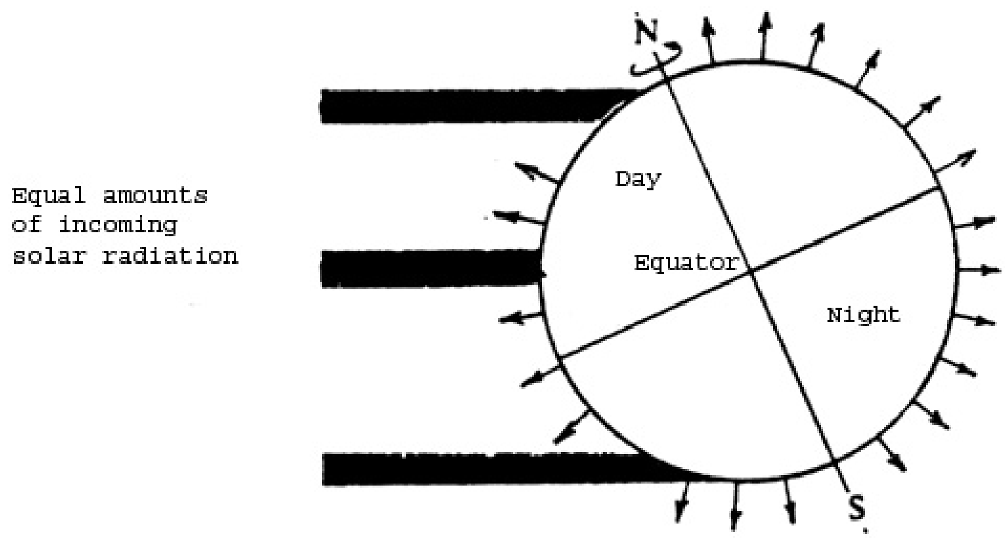

Figure 1.7. A depiction of the basic elements of the annually-averaged Earth System heat budget; (1) incoming solar radiation, which is unevenly distributed due to geometric factors, and (2) the is more evenly distributed outgoing long wave radiation.

Geometrical spreading accounts for much of the non-uniformity. Figure 1.8 shows how the same amount of solar radiant energy is spread over a greater area in the higher latitudes relative to the equatorial latitudes. Hence the incoming energy flux at the outer edge of the atmosphere is less in the more polar latitudes than the tropic al latitudes.

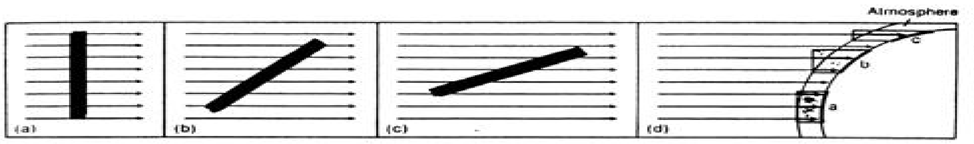

Figure 1.8. Equal amounts of solar energy at (a) are spread over increasingly larger areas at more polar latitudes as illustrated in (b) and (c) respectively. (Duxbury & Duxbury, 1984)

However the distribution of outgoing long wave energy is much more even over the earth at the edge of the atmosphere than is that of solar radiation energy. Consequently there is a surplus of incoming solar energy at the edge of the atmosphere in the equatorial regions and deficit of incoming solar energy to supply the long wave radiation demand.

The amount of heat flux reaching different areas of the Earth will also be affected by the local length of day. The actual amount of heat flux which eventually reaches the Earth's surface will be affected by other factors, including the amount of (1) absorption by dust; (2) cloud absorption and scatter and (3) surface reflection due to ice cover, etc.

The overall heat budget of the Earth at the surface varies with latitude in a way shown in Figure 1.9. The thermal energy imbalances implied by in Figure 1.9 are the basis for the poleward heat transport by the atmosphere/ocean system (Figure 1.10). The combined ocean and atmospheric circulation (weather) result from this thermal energy gradient.

Most of the direct solar energy is absorbed initially by the land and oceans. Because of the ocean's extensiveness (71% of the Earth surface) and relatively high heat capacity, it is the principal heat reservoir in the Earth system. The atmosphere with its relatively low heat capacity is principally a conveyor of heat from the equatorial regions to the polar regions. The exchange of heat between the ocean and atmosphere is affected by a complex suite of heat transfer processes. Heat from the ocean causes atmospheric winds, which in turn cause ocean currents. The winds and the oceans partner to move heat poleward.

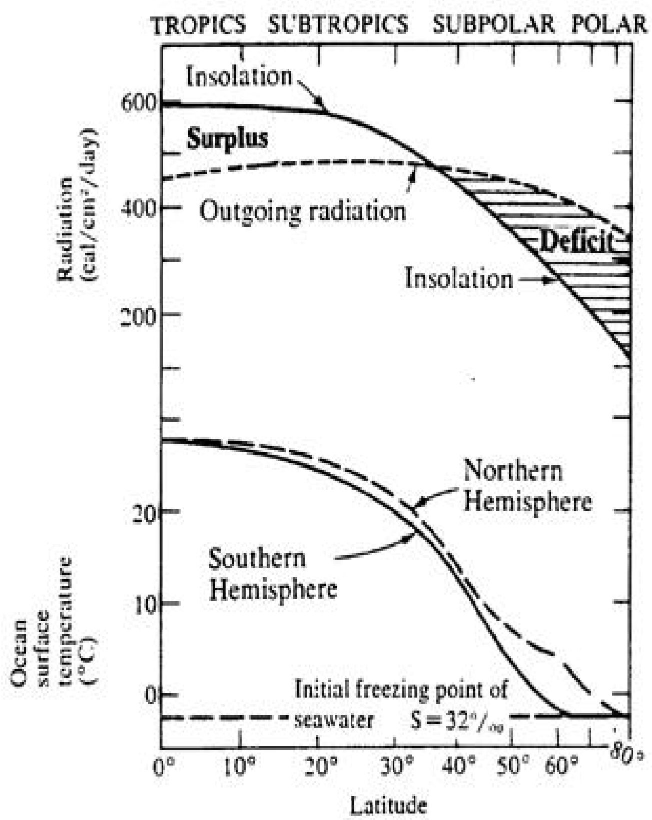

Figure 1.9. (a) Incoming (solar insulation) and outgoing heat flux as a function of latitude, (b) Ocean surface temperature distributions.

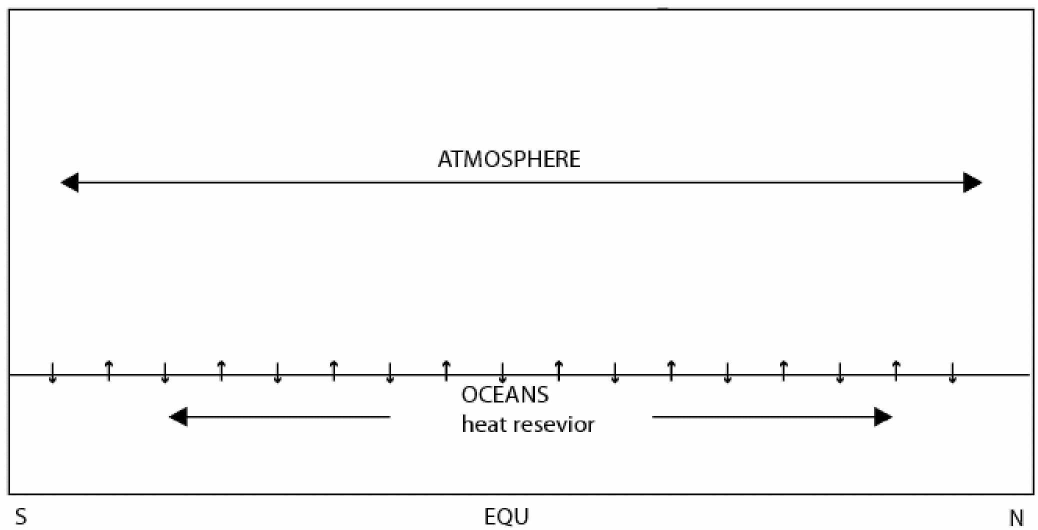

Figure 1.10. Poleward heat transport via the atmosphere/ocean partnership.

The details of the processes that couple the ocean and the atmosphere are important to understanding the climate dynamics of the Earth as a whole; as well as the ocean environment itself. A discussion of the details of these important heat transfer processes that combine in the smooth operation of the PLANETARY HEAT ENGINE begins with the series of definitions that follow.

HEAT ENERGY - The amount of energy contained in all molecular motions within the particular fluid, gas or solid. Heat or thermal energy is measured in units of *gm-calories* (or more commonly *calories*) which are defined by:

1 gm-cal will raise the temperature of 1 gram of distilled water by $1^{o}C$.

Because thermal energy is just one form of energy, it can be converted to the usual measures of mechanical energy according to following:

$$1\,\text{gm- calorie} = 4.183 \text{ joules } (\text{kg m}^2/\text{sec}^2)$$
$$= 4.183 \times 10^7 \text{ ergs } \left(\frac{\text{gm cm}^2}{\text{sec}^2}\right)$$

This means that the frictional process of stirring 4.183 x 10^7 ergs of mechanical energy into 1 gm of water will raise it's temperature by 1^oC.

SPECIFIC HEAT or HEAT CAPACITY (C_p) - The amount of heat energy required to raise 1 gm of material by 1^oC, when working at constant pressure.

$$C_p \text{ for seawater is about } (\sim)\ 0.95 \pm .05 \text{ cal/gm/ } {}^\circ\text{C},$$

The specific heat of water, which depends weakly on salinity (S), temperature (T) and pressure (p), is relatively high in comparison to other common Earth materials , for which $C_p \cong 0.2$ cal/gm/ $^\circ$C.

HEAT FLUX (Q) - The amount of heat energy passing through a unit area in a unit time. Heat flux is measured in units of cal/cm^2/sec (or langleys / sec or ly/sec).

The effect of heat flux on the temperature changes ΔT in a laterally infinite slab of material with thickness Δh, density r specific heat at constant pressure C_p (see Figure 1.11) over a time increment Δt, according to

$$\Delta T = \frac{Q_{in} - Q_{out}}{\Delta h} \frac{1}{r\, C_p} \Delta t,$$

where Q_{in} and Q_{out} are total incoming and outgoing heat fluxes respectively.

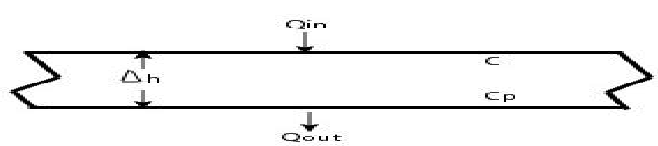

Figure 1.11 A laterally semi-infinite slab of material with incoming and outgoing heat fluxes.

We do this in the ocean where there are several kinds of heat fluxes between the ocean surface and the atmosphere (see Figure 1.12), namely

Q_s - incoming short wave radiation

Q_b - outgoing long wave re-radiation (or back-radiation)

Q_e - outgoing latent heat flux

Q_h - outgoing sensible heat flux

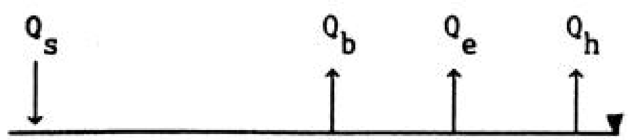

Figure 1.12. The different heat flux components at the sea surface.

Both Q_s and Q_b are radiative heat fluxes which can be characterized the set of physical laws that are considered in the following digression.

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Digression: Radiation Laws for a Black Body

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The <u>total amount of heat flux</u> radiated from a black body can be estimated according to the <u>Stefan-Boltzmann Law</u> , which is

$$Q = \sigma T^4 \qquad ,$$

where T is the absolute temperature of the radiating body in degrees Kelvin (^oK)

which is

$$°K = °C + 273.15 \quad ;$$

and s is the Stefan-Boltzmann constant with a value of

$$s = 1.36 \text{ x } 10^{-12} \text{ cal/ cm}^2\text{- sec-}°K^4 .$$

The maximum wavelength at which the heat flux is radiated from a body can be estimated according the Wien Displacement Law, which is

$$l_{max}\, T = 2.9 \text{ x} 10^{-1} \text{ cm } °K,$$

where l_{max} is the wavelength of the peak of the *energy spectrum* as shown in Figure 1.13.

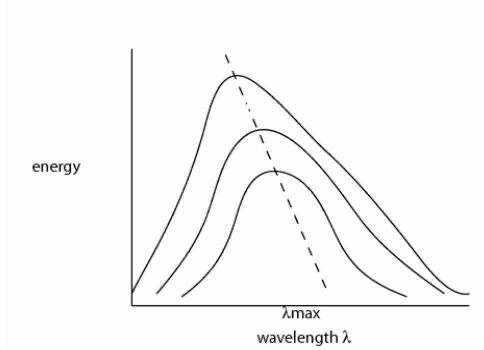

Figure 1.13. The family of energy spectra for radiation leaving bodies of differing absolute temperatures. The Wien Displacement Law (dashed line) defines the wavelengths ($?_{max}$) associated with the peaks of different spectra.
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RADIATIVE HEAT FLUX COMPONENTS

The nature of solar and long wave radiation heat fluxes are considered in terms of the Stefan-Boltzmann and the Wien Displacement (W-D) radiation laws.

Solar Radiation (Q_s): The effective surface temperature of the sun is *5800°C*. So according to the W-D law, its characteristic wavelength is $l_{max} = 0.54$ microns (1 micron = µ = 10^{-6} m), with 99% of the energy at wavelengths $l < 4$µ. Thus $\mathbf{Q_s}$ is *short wave radiation* (typically 1.54 x 10^3 cal/cm^2/sec) impinging on the Earth. This quantity is sometimes referred to as *solar insolation.*

Back Radiation (Q_b): The Earth also radiates energy back into space, although its absolute temperature is much lower that that of the sun. The effective surface temperature of the Earth is 290 ^oK. So according to W-D $\lambda_{max} = 10$ microns , with 90% of the energy in the 3 to 80μ wavelength band. Thus $\mathbf{Q_b}$ departs from the Earth's surface as *long wave radiation*. The the typical net oceanic $\mathbf{Q_b}$ value is 0.96 x 10^{-2} cal/cm^2/sec, depending on sea surface temperature as well as the water vapor content of the air above.

NON-RADIATIVE HEAT FLUX COMPONENTS

Latent and sensible heat fluxes are due to complex air-sea interaction processes and are estimated using *empirically-derived* bulk formula based on experimental observations.

Latent Heat Flux (Q_e): Heat, drawn from the local ocean environment, is required to evaporate water - that is provide enough energy to convert water molecules at the surface from liquid to gas ...and thus break away from the ocean surface. *Latent heat flux* is associated with this process and can be estimated by

$$Q_e = L \times F,$$

where **F** is the mass flux of water evaporated from the sea surface and **L** (cal/gm) is the *latent heat of sea water*

$$L = [596 - .52\, T_s(°C)].$$

which depends on sea surface temperature T_s .

F is difficult to measure, so it is usually estimated from the "bulk relation"

$$F\left(\frac{gm}{cm^2\text{-}day}\right) = .014(e_s - e_a)\,W\,,$$

where **W** (m/sec) is the wind *speed* at 10 m elevation, $\mathbf{e_s}$ (millibars; mb) is the *saturated water vapor pressure* at the sea surface temperature $\mathbf{T_s}$ ($\mathbf{e_s}$ = 0.98 x the saturated vapor pressure of distilled water e_d -shown in Figure 1.14); $\mathbf{e_a}$ (mb) is the *water vapor pressure* at 10 m above the sea surface based on measured (a) *air temperature* Ta and (b) *relative humidity* RH. Estimating water vapor pressure and RH are addressed in the following digression.

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Digression - Water Vapor Pressure and Relative Humidity

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Water vapor pressure (or partial pressure) is the portion of the total air pressure caused by the water vapor. The phase diagram for water (Figure 1.14) relates the saturated vapor pressure to temperature.

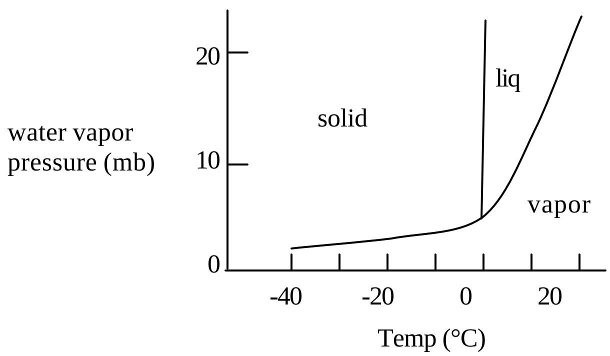

Figure 1.14. Phase diagram showing distilled water vapor pressure e_d. The line dividing liquid and vapor is the saturated water vapor pressure.

Relative Humidity (RH) is the ratio of the water vapor pressure of a parcel of air e_a to the saturated vapor pressure e_{as} at the same specified temperature or

$$RH = \frac{e_a}{e_{as}}$$

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Sensible Heat Flux: Q_h, is the combined transfer of heat due to conduction and forced convection. It can be estimated using measured quantities and the Bowen ratio, R, according to

$$R = \frac{Q_h}{Q_e} = 0.64 \frac{T_s - T_a}{e_s - e_a},$$

where T_s, e_s are at sea surface values and T_a, e_a are values at 10 m elevation respectively.

HEAT BUDGET OF THE OCEAN

In considering the heat budget of the ocean, the sources and sinks of heat flux must be identified.

Sources:

- Q_s; short wave radiation from sun and diffuse skylight
- Q_b; long wave radiation from the atmosphere
- Q_h; sensible heat transfer from the atmosphere by conduction
- Q_e; latent heat transfer by water condensation on the sea surface

Sinks:

- Q_b; long wave radiation loss from the sea surface

Q_h; sensible heat loss by conduction

Q_e; latent heat loss through evaporation of surface water

The relatively complicated picture of incoming and outgoing heat fluxes (Figure 1.15). represents an annual- and global averaged picture in which all heat fluxes are expressed as a percentage of the total incoming solar heat flux.

Notes:

(1) Only about ½ of the incoming solar radiation reaches the sea surface and only ½ of that directly from the sun.

(2) Only 5% of the long wave radiation leaving the sea surface escapes directly to space. The rest is absorbed by the H_2O/CO_2 rich atmosphere; 114% vs. 16% due to short wave solar radiation.

(3) The atmosphere reradiates a significant proportion of the long wave (or infrared) radiation back to the sea surface where it is absorbed and reradiated.

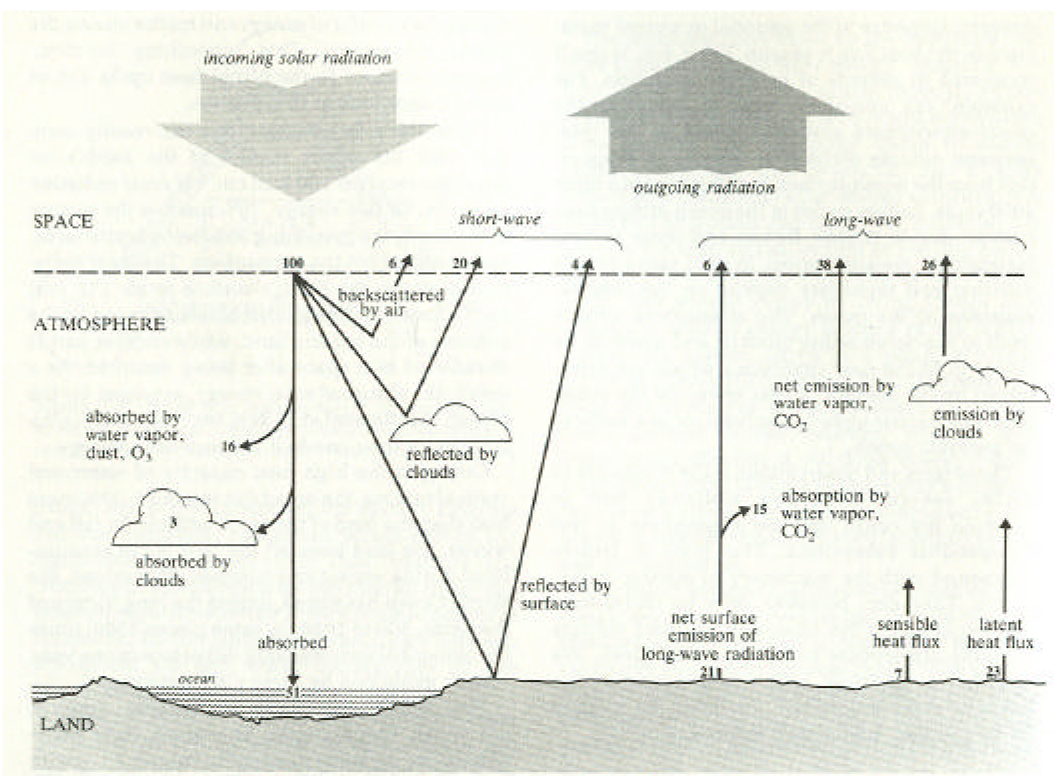


Figure 1.15. The mean annual radiation and heat balance of the atmosphere, relative to 100 units

of incoming solar radiation, based on satellite measurements and conventional observations. (Tolmazin, 1985)

This cycling of long wave radiation is the basis of the so-called GREENHOUSE EFFECT. (Actually green houses do not trap heat in this way.) A warmer atmosphere produces shorter wavelength back radiation - thus escaping absorption by "greenhouse" gases (see Figure 1.16).

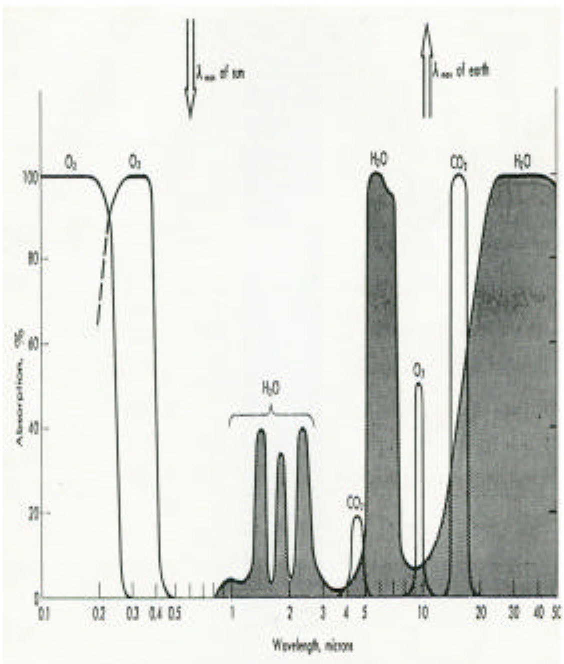


Figure 1.16. Radiation spectra of sun and earth and absorption spectra of the atmosphere. (Goody & Robinson, 1951)

As a function of latitude the ocean heat budget looks something like what is pictured in Figure 1.17.

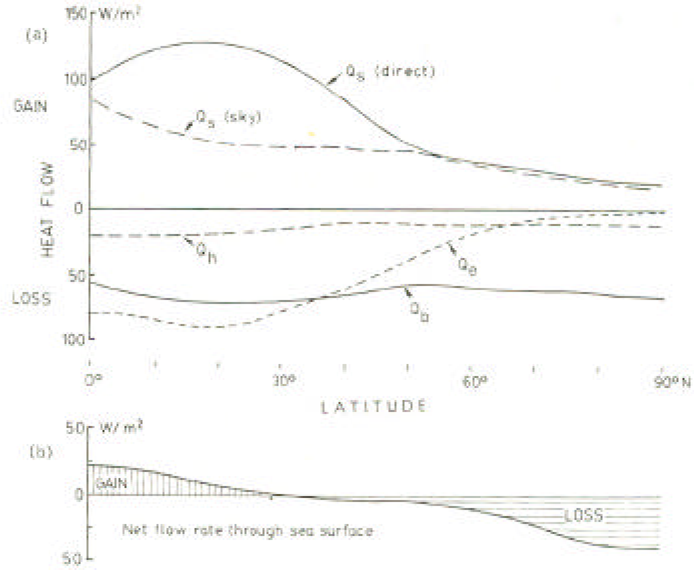


Figure 1.17. The latitudinal distribution of the different heat flux components. (Pickard & Emery, 1982)

The ocean, like the earth as a whole, also has a surplus of heat in the equatorial regions and a deficit in the polar regions. The heat surplus must be transported away from the equatorial zones by ocean currents to achieve the local balance. The amount of heat transported at each latitude can be determined by considering a latitudinal band of ocean between latitude lines as shown in Figure 1.18.

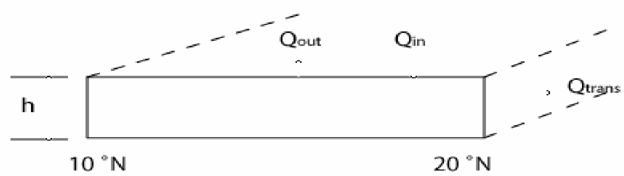


Figure 1.18. Schematic for estimating net meridional heat transport in the 10°N to 20°N latitudinal band.

The corresponding heat balance for a particular latitude band is

$$Q_{in} - Q_{out} - Q_{trans} = r\,Cp\,\Delta h\,\frac{\Delta T}{\Delta t},$$

$$\text{where } Q_{in} = Q_{s\,total}$$

$$Q_{out} = Q_{b\,net} + Q_{e\,net} + Q_h\,.$$

Since on average $\Delta T \sim 0$, Q_{trans} can be determined from the distributions of Q_s, Q_e, Q_b, Q_h as shown in Figure 1.17. Thus we can derive the distribution of the poleward transport of heat across each line of latitude for the Earth as a whole, the ocean and the atmosphere separately (Figure 1.19).

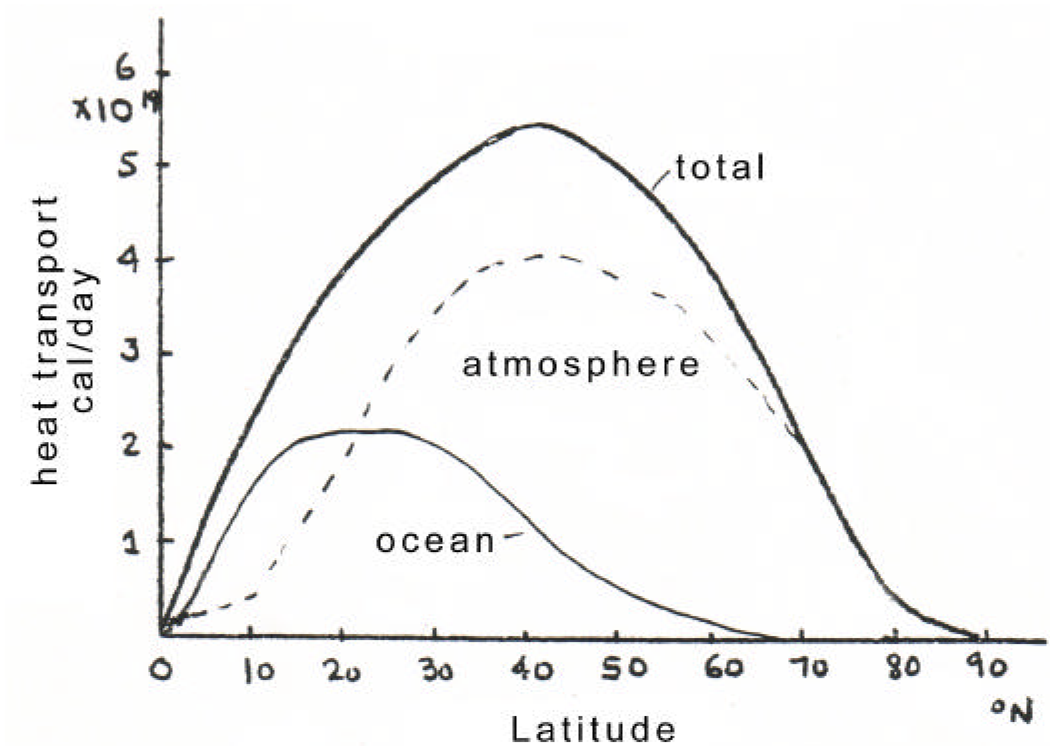
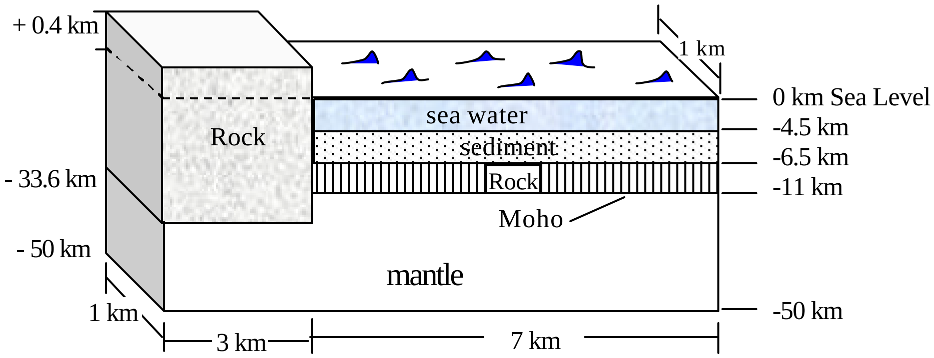


Figure 1.19. Poleward heat transport distribution for the Earth (total), atmosphere and oceans in the Northern Hemishere. (After Vonder Haar and Oort, 1973).

Chapter 1 - PROBLEMS

Problem 1.1 Isostasy – The Weathering Problem

Consider the model earth below with material densities (gm/cm^3) are; Rock ρ_R = 2.85; mantle ρ_M = 3.27; sea water ρ_{SW}= 1.03; sediments - ρ_S = 2.77.

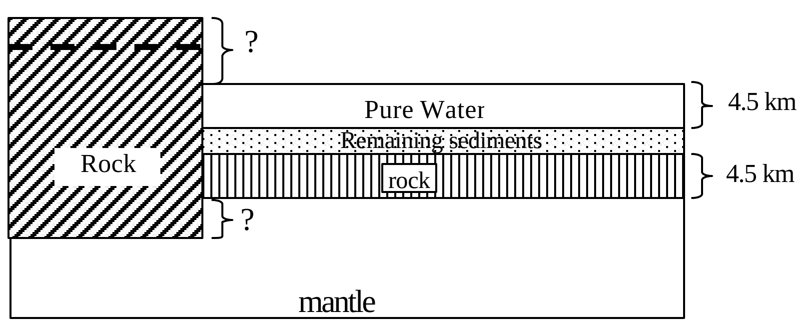


a) Show the system is in isostatic equilibrium, i.e., that the pressure at 50 km depth is equal under the continents and the ocean.

b) For each kilogram of sea water 600 grams of rock were weathered in the past. Calculate the total number of grams of rock weathered. What is the volume of rock in cm^3 and km^3?

c) The salts in the weathered rock have been dissolved and the rest has settled to the sea floor as sediments. Now for each kilogram of sea water remove 30 gm of salts leaving pure water (density = 1.00 gm/cm^3) and 570 gms of sediment from the sea floor. What is the volume of sediment removed in km^3?

d) Placing all of this mass as rock on the continents yields a system that appears as



Conserving the amount of mantle rock, adjust the system to isostatic equilibrium. What is the elevation of the unweathered continent relative to sea level? What is the depth of the bottom of the continent relative to bottom of the ocean crustal rock?

Problem 1.2 Ocean Volume

Given that (a) approximately 70% of the Earth is covered by the oceans, (b) the mean depth of the world's oceans is approximately 4000 m; and (c) the mean radius of the Earth is 6000 km., calculate the percentage of the Earth's volume that is comprised by the oceans. Show all of your work including a diagram of the problem.

Problem 1.3 Solar Heating – The Greenhouse Effect

(a) The sun (radius R_S) radiates energy uniformly in all directions at a temperature T_S. If a spherical planet of radius R is at a distance d from the sun, how much energy does it intercept in terms of T_S, R_S, R, d and the Stephan-Boltzmann constant σ ?

(b) Suppose the planet is perfectly heat conducting and is black so that it is uniform temperature. If the planet radiates away the same amount of energy that it receives from space, then what must its temperature be in terms of the variables in part (a)? Now compute this temperature assuming the planet is the Earth using

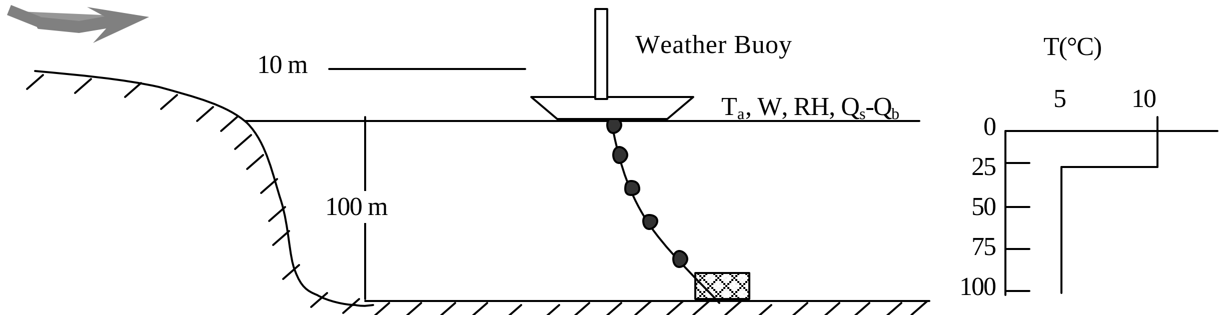
T_S	$= 5800°K$	$d = 150 \times 10^6$ km
R_S	$= 6.9 \times 10^5$ km	$R = 6371$ km

(c) Suppose 1/2 of the solar heat flux is reflected from the Earth and 1/2 is absorbed and then re-radiated. Then what would the Earth's temperature T be?

d) Explain why the Earth's surface is warmer than the temperature in part (c).

e) Suppose only 40% of the radiation radiated by the Earth in case (c) can escape. What is the temperature at the surface necessary for a radiation balance? Suppose by adding CO_2 to the atmosphere, the window opening decreases by 2% so that only 39% of the radiation can escapes. What is T under that scenario?

Problem 1.4 Air-Sea Heat Transfer



Consider the situation depicted in the figure above where the generally cold dry wind blows offshore along the coast of Maine during the winter. An offshore weather buoy measures the air temperature, T_a, the wind speed, W, the relative humidity, RH, and the air pressure, P_a, at an elevation of 10m. In addition, a radiometer (chapter 6 in Pickard) is used to provide the net radiative heat flux to the sea surface (Q_s-Q_b). An array of thermistors (black dots) are attached to the mooring line of the weather buoy to measure water column temperature time series, which are averaged to obtain the average temperature profile (shown to the right) for the day in question. Given the daily averaged values of T_a = -10°C, W = 20 ms^{-1}, RH = 0.30, and Qs-Qb = 100 $cal/cm^2/day$, P_a = 1000 mb and the assumption that the saturated water vapor pressure over distilled water e_d, can be expressed as

$$e_d(\text{mb}) = 0.61 \times T(°\text{C}) + 6.1 \qquad \text{for } 0° \le T \le 15°\text{C}$$

$$\text{and } e_d(\text{mb}) = 0.32 \times T(°\text{C}) + 6.1 \qquad \text{for } -15°\text{C} \le T \le 0°\text{C}$$

(a) What is the average amount of water (per unit surface area) evaporated this day? What is the latent heat flux associated with this mass transfer?

(b) What is the average sensible heat flux during this day?

(c) What is the average <u>net</u> heat flux from the sea surface during this day?

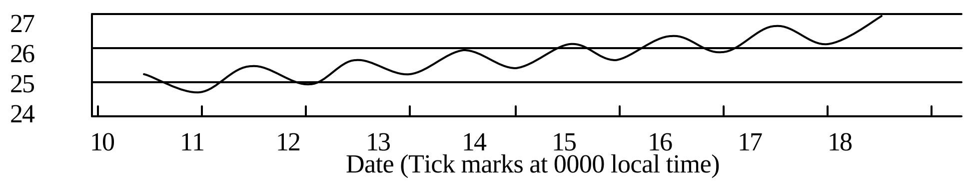
(d) Assume the upper 25 m of the ocean is well mixed and at 10°C. If the same amount of heat in (c) were to be transferred each successive day and the upper 25 meters of the ocean were to remain well mixed, what would be the water temperature after three days?

(e) Assume that water density is determined solely by temperature. What do you expect would happen to the water column if this cooling process were to continue?

Problem 1.5 Daily Heating - Ocean

a) Suppose the heat flux at the surface of the ocean is 1/4 ly/min. What is the change in temperature if this heating continues through 12 hours and is distributed through the upper 10 meters? -- upper 100 meters?

b) Below is a time series record of the upper ocean temperature T (in ^{o}C) with a diurnal (i.e.daily) variability superposed on a secular increase. This record



suggests that the daily net air-to-sea heat flux could be modeled as

$$Q = Q_o \sin(2\pi\ t / t_o) \qquad ,$$

where t_o = 24 hr.

(a) Assuming that the heat entering the water is evenly distributed over the mixed layer depth of H, what is the equation for the temperature of the layer as a function of time?

[Hint: From your class notes $\rho C_p \Delta T = Q \Delta t / H$...or $\Delta T / \Delta t \sim dT/dt = Q/(\rho C_p H)$]

(b) If the amplitude (peak to peak) of the sea surface temperature change is 1°C and Q_o is 1 ly/min, then what is the depth H of the mixed layer?

